all of that which is *other than a*—that is, if everything must be one or the other (a or b) and if nothing can be both. These two properties constitute the definition of a pair of contradictories (whether terms or propositions), namely, they are mutually exclusive, and they are together exhaustive; expressed in the language of 'exact logic,' these properties are (writing $\bar{x}$ for the negative of x and $+$ for *or*):

(1) $x\bar{x} < 0$,	(2) $\infty < x + \bar{x}$,
what is at once x and $\bar{x}$ does not exist, or, in the language of propositions, the conjoint occurrence of x and $\bar{x}$ does not take place.	everything is either x or $\bar{x}$, or, in the language of propositions, what can occur is either x or $\bar{x}$, or, reality entails x or $\bar{x}$—there is no *tertium quid*.

Together these properties constitute the requirements of contradiction or of exact negation; it is a very inelegant piece of nomenclature (besides that it leads to actual confusion) to refer to (1) alone as the 'principle of contradiction.' Better names for them are (1) exclusion and (2) exhaustion (in place of excluded middle). In the common phraseology we are obliged to commit the absurdity of saying that two terms or propositions may satisfy the 'principle of contradiction' and still not be contradictory (since they may lack the quality of being exhaustive). The mere fact that (1) has been called the principle of contradiction has given it a pretended superiority over the other which it by no means deserves; they are of equal importance in the conducting of reasoning processes. In fact, for every formal argument which rests upon (1) there is a corresponding argument which rests upon (2): thus in the case of the fundamental law of TRANSPOSITION (q. v.), which affirms the identity of these two propositions, (m) the student who is not a citizen is not a voter; (n) every student is either a citizen or not a voter; that (m) follows from (n) depends upon one of these principles, and that (n) follows from (m) depends upon the other. These two names, exhaustion and exclusion, have the great advantage that they permit the formation of adjectives; thus we may say that the test for the contradictoriness of two terms or propositions which are not on their face the negatives one of another is that they should be (1) mutually exclusive and (2) together exhaustive.

It may be noticed that if two terms are exhaustive but not exclusive, their negatives are exclusive but not exhaustive. Thus within the field of number, 'prime' and 'even' are exclusive (no number can be both) but not exhaustive (except in the limiting case of two, some numbers can be neither), while 'not even' and 'not prime' are exhaustive and not exclusive.

In the case of propositions, 'contrary' and 'subcontrary' are badly chosen names for the OPPOSITION (q. v.) of A and E, O and I, respectively, of the traditional logical scheme; they do not carry their meaning on their face, and hence are unnecessarily difficult for the learner to bear in mind. A and E should be said to be mutually exclusive (but not exhaustive), O and I to be conjointly exhaustive (but not exclusive). This relation of qualities is then seen to be a particular case merely of the above-stated general rule.

Again, 'no a is b' and 'all a is b' are exclusive but not exhaustive, while 'some a is b' and 'some a is not b' are exhaustive but not exclusive (provided in both cases that a exists).

Laws of thought is not a good name for these two characteristics; they should rather be called the laws (if laws at all) of negation. Properly speaking, the laws of thought are all the rules of logic; of these laws there is one which is of far more fundamental importance than those usually referred to under the name, namely, the law that if a is b and b is c, it can be concluded that a is c. This is the great law of thought, and everything else is of minor importance in comparison with it. It is singular that it is not usually enumerated under the name. Another law of thought of equal consequence with those usually so called is, according to Sigwart, the law that the double negative is equivalent to an affirmative, $\bar{\bar{x}} = x$, or

(3) $x < \bar{\bar{x}}$,	(4) $\bar{\bar{x}} < x$.

But these are not fundamental, for from the principles of

Exclusion,	Exhaustion,
(1) $x\bar{x} < 0$,	(2) $\infty < x + \bar{x}$,

it follows

by (2) that	by (1) that
$x < \bar{\bar{x}}$,	$\bar{\bar{x}} < x$.

(C.L.F.)

Literature: for the history of these principles see UEBERWEG, Syst. d. Logik, §§ 75–80; PRANTL, Gesch. d. Logik (see 'principium' in the indices to the four volumes). There are additional notes in an appendix to HAMILTON, Lects. on Logic. (C.S.P.)

END OF VOL. I.

"The Century's Great Men in Science."
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The CENTURY'S

GREAT MEN IN SCIENCE

By CHARLES. S. PEIRCE

How shall we determine that men are great? Who, for instance, shall we say are the great men of science? The men who have made the great and fruitful discoveries? Such discoveries in the nineteenth century have mostly been made independently by two or more persons. Darwin and Wallace simultaneously put forth the hypothesis of natural selection. Clausius, Rankine, and Sadi-Carnot, perhaps Kelvin, worked out the mechanical theory of heat. Krönig, Clausius, Joule, Herapath, Waterston, and Daniel Bernouilli independently suggested the kinetical theory of gases. I do not know how many minds besides Robert Mayer, Colding, Joule, and Helmholtz hit upon the doctrine of the conservation of energy. Faraday and Joseph Henry brought magneto-electricity to light. The pack of writers who were on the warm scent of the periodic law of the chemical elements approached two hundred when the discovery itself, a most difficult inference, was partly achieved by Lothar Meyer, wholly by Mendeléef. When great discoveries were thus in the air, shall that brain necessarily be deemed great upon which they happened earliest to condense, or the man supereminent who, by the unmeaning rule of priority of publication, gets the credit in brief statements? No, this method of estimation, natural as it is to make success the standard of measure, will not do.

Shall we, then, by a logical analysis, draw up an abstract definition of greatness, and call those men great who conform to it? If there were no dispute about the nature of greatness, this might probably prove the most convenient plan. It would be like a rule of grammar adduced to decide whether a phrase is good English or not. Nor would the circumstance that the definition could not be as explicit and determinate as a rule of grammar constitute a serious difficulty. Unfortunately, however, among the few writers who have seriously studied the question, the most extreme differences prevail as to the nature of great men. Some hold that they are fashioned of the most ordinary clay, and that only their rearing and environment, conjoined with fortunate opportunities, make them what they are. The heaviest weight, intellectually, among these writers, maintains, on the other hand, that circumstances are as powerless to suppress the great man as they would be to subject a human being to a nation of dogs. But it was only the blundering Malvolio who got the notion that some are born great; the sentence of the astute Maria was: "Some are become great, some atcheeves greatnesse, and some have greatnesse thrust uppon em." Amid this difference of opinion,

any definition of greatness would be like a disputed rule of grammar. Just as a rule of grammar does not render an expression bad English, but only generalizes the fact that good writers do not use it, so, in order to establish a definition of greatness, it would be necessary to begin by ascertaining what men were, and what men were not, great; and, that having been done, the rule might as well be dispensed with. My opinion will, I fear, be set down by some intellectual men as foolishness, though it has not been lightly formed, nor without long years of experimentation--that the way to judge of whether a man was great or not is to put aside all analysis, to contemplate attentively his life and works, and then to look into one's heart and estimate the impression one finds to have been made. This is the way in which one would decide whether a mountain were sublime or not. The great man is the impressive personality; and the question whether he is great is a question of impression.

The glory of the nineteenth century has been its science; and its scientific great men are those whom I mean here to consider. Their distinctive characteristic throughout the century, and more and more so in each succeeding generation, has been devotion to the pursuit of truth for truth's sake. In this century we have not heard a Franklin asking, "What signifies a philosophy which does not apply itself to some use?"--a remark that could be paralled by utterances of Laplace, of Rumford, of Buffon, and of many another well-qualified spokesman of eighteenth-century science. It was in the early dawn of the nineteenth that Gauss (or was it Dirichlet?) gave as the reason of his passion for the Theory of Numbers that "it is a pure virgin that never has been, and never can be, prostituted to any practical application whatsoever." It was my inestimable privilege to have felt as a boy the warmth of the steadily burning enthusiasm of the scientific generation of Darwin, most of the leaders of which at home I knew intimately, and some very well in almost every country of Europe. I particularize that generation without having any reason to suspect that that flame has since burned dimmer or less purely, but simply because if a word belonged to one's mother-tongue, one may be supposed to know unerringly the meaning the teachers of one's boyhood attached to it.

The word science was one often in those men's mouths, and I am quite sure they did not mean by it "systematized knowledge," as former ages had defined it, nor anything set down in a book, but, on the contrary, a mode of life; not knowledge, but the devoted, well-considered life-pursuit of knowledge; devotion to Truth--not "devotion to truth as one sees it," for that is no devotion to truth at all, but only to party--no, far from that, devotion to the truth that the man is not yet able to see but is striving to obtain. The word was thus, from the etymological point of view, already a misnomer. And so it remains with the scientists of today. What they meant, and still mean, by "science" ought, etymologically, to be called philosophy. But during the nineteenth century it was only a metaphysical professor of a now obsolescent type, as I hope, who could sit in his academic chair, puffed up with his "systematized knowledge"--no true philosopher, but a mere philodoxer. For a snapshot at the nineteenth-century man of science, one may take Sir Humphrey Davy, willing, as early as 1818, seriously to investigate the liquefaction of the blood of St. Januarius; or John Tyndall, with scientific ingenuousness proposing that prayer-test to which no clerical Elijah has yet been found with the faith and good faith to respond; or William Crookes, devoting years of his magnificent powers to examining the supposed evidences of the direct action of mind upon matter,

in the face of the world's scorn. Contrast these instances with the refusal of Laplace and Biot in the closing years of the previous century, to accept the evidence that stones fall from heaven (evidence proving that they do so daily), simply because their prepossessions were the other way. One of the geologist brothers De Luc declared that he would not believe such a thing though he saw it with his own eyes; and a scientifically given English ecclesiastic who happened to be sojourning in Siena when a shower of aerolites were dashed in broad daylight into an open square of that town, wrote home that having seen the stones he had found the testimony of eye-witnesses so unimpeachable and so trustworthy that--that he accepted the fact, you will say? by no means--that he knew not what to think! Such was the bon sens that guided the eighteenth century--a pretty phrase for ineradicable prejudice.

To this self-effacement before the grandeur of reason and truth is traceable the greatness of nineteenth-century science, most obviously in mathematics. In the minds of eighteenth-century mathematicians their science existed for the sake of its applications. Forgetfulness of this was in their eyes reprehensible, immoral. The question was, what would a given piece of mathematics do? They liked smooth-running and elegant machinery--there was economy in that; but they were not sedulous that it should have symmetry; idle admiration of its beauty they hardly approved. If it was excessively complicated and intricate, that was regarded rather as a feature to be proud of than as a blemish. Were the complete revolution that the nineteenth century wrought upon the ideal of mathematics not notorious, one could soon convince himself of it by looking over almost any modern treatise--say, Salmon on 'Higher Plane Curves.' That volume, for example, would be found replete with theorems hardly any of which hold good for any curves that could really exist. Realizable curves have hardly been studied at all, for the reason that they do not yield a beautiful theory such as is now exacted. Modern mathematics is highly artistic. A simple theme is chosen, some conception pretty and charming in itself. Then it is shown that by simply holding this idea up to one's eye and looking through it, a whole forest that before seemed a thick and tangled jungle of bushes and briers is seen to be in reality an orderly garden. The word generalization really cannot be fully understood without studying modern mathematics; nor can the beauty of generalization be in any other way so well appreciated. There is here no need of throwing out "extreme cases." Far from that, it is precisely in the extreme cases that the power and beauty of the magic eyeglass is most apparent and most marvellous. Let me take back the word "magic," though; for the reasonableness of it is just its crowning charm. I must not be led away from my point, to expatiate upon the reposefulness of the new mathematics, upon how it relieves us of that tiresome imp, Man, and from the most importunate and unsatisfactory of the race, one's self. Suffice it to say that it is so reasonable, so simple, so easy to read, when the right view has once been attained, that the student may easily forget what arduous labors were expended in constructing the first convenient pathway to that lofty summit, what mastery over intricacies, far beyond that of the eighteenth-century master. "It must not be supposed," said C. G. J. Jacobi, one of the simplifying pioneers, "that it is to a gift of nature that I owe such mathematical power as I possess. No, it has come by hard work, hard work. Not mere industry, but brain-splitting thinking--hard work; hard work that has often endangered my health." Such reflections enable us to perceive that if

modern mathematics is great, so also were the men who made it great.

The science next in abstractness after mathematics is logic. The contributions of the eighteenth century to this subject were enormous. In pure logic, the doctrine of chances, which has been the logical guide of the exact sciences and is now illuminating the pathway of the theory of evolution, and is destined to still higher uses, received at the hands of Jacob Bernouilli and of Laplace developments of the first importance. In the theory of cognition, Berkeley and Kant laid solid foundations; their personal greatness is incontestable. This is hardly true of Hume. In the nineteenth century, Boole created a method of miraculous fruitfulness, which aided in the development of the logic of relatives, and threw great light on the doctrine of probability, and thereby upon the theory and rules of inductive reasoning. De Morgan added an entirely new kind of syllogism, and brought the logic of relatives into existence, which revolutionizes general conceptions of reasoning. The works of Comte, Whewell, J. S. Mill, Jevons, and others upon the philosophy of inductive science were less successful or fruitful. In the more metaphysical part of logic, the philosophy of Hegel, though it cannot be accepted on the whole, was the work of a great man. In metaphysics and general cosmology, the attitude of the century has been expectant. Herbert Spencer has been proclaimed as a sort of scientific messiah by a group of followers more ardent than philosophic, which does not seem to be gathering strength.

At the head of the physical sciences stands nomological physics. Dr. Thomas Young was here the earliest great man of the century, whose intellect illuminated every corner to which it was directed, taking the first difficult steps in the decipherment of the hieroglyphics, originating the doctrine of color-mixtures, propounding the correct theory of light, and illuminative everywhere. It gives a realizing sense of the century's progress that this great man in its early years should have opined that experimentation in general had then been pushed about far enough. On that occasion, it was not his usual logic, but the eighteenth-century watchword "le bon sens," that was his guide, with the sort of result it is continually turning out when used beyond its proper sphere of every-day practical affairs. The advance of years with their experience has led physicists to expend more and vastly more effort upon extreme precision, against every protest of good sense. What has come of it? Marconi's wireless telegraphy, for one thing. For it was the precision with which the velocity of light on the one hand, and the ratio of statical and dynamical constants of electricity on the other, had been determined that proved to Maxwell that the vibrating medium of light was the substance of electricity, a theory that his great follower Hertz applied to making giant light waves less affected by obstructions than even those of sound. I dare say sapient "good sense" pooh-poohs those wonderful new substances, helium and the rest, that seem the connecting link between ordinary matter and the ether. So it would be useless to point out that their discovery was entirely due to Lord Rayleigh's fastidiousness in the determination of the destiny of nitrogen. But it has to be noted as a characteristic of the great physicists of the nineteenth century that their reverence for every feature of the phenomenon, however minute, has been in thorough disaccord with the older "good sense." The greatest advances in physics during the century were made by several men at once. Certain ideas would come somehow to be

in the air; and by the time they had crystallized for a student here and there, he would hesitate to announce as original conceptions what he had reason to suppose many men shared, while he knew that the larger body would not be yet ready to accept them. Under those circumstances, priority of publication can signify nothing except haste.

Of all men of the century Faraday had the greatest power of drawing ideas straight out of his experiments and making his physical apparatus do his thinking, so that experimentation and inference were not two proceedings, but one. To understand what this means, read his 'Researches on Electricity.' His genius was thus higher than that of Helmholtz, who fitted a phenomenon with an appropriate conception out of his store, as one might fit a bottle with a stopper. The most wonderful capacity for "catching on" to the ideas of nature when these were of a complicated kind was shown by Mendeléef in making out the periodic law of the chemical elements, as one might make out the meaning of a pantomime, from data so fragmentary, and in some cases erroneous, that the interpretation involved the correction of sundry facts, corrections since confirmed, as well as the prediction of the very peculiar properties of the unknown gallium, scandium, and germanium, which were soon afterward actually met with. Minute examination of all his utterances convinces one that Mendeléef's mental processes in this unparalleled induction were largely sub-conscious and, as such, indicate an absorption of the man's whole being in his devotion to the reason in facts.

A great naturalist, as well as I can make out, is a man whose capacious skull allows of his being on the alert to a hundred different things at once, this same alertness being connected with a power of seeing the relations between different complicated sets of phenomena when they are presented in their entirety. The eighteenth century had its Linnaeus, whose greatness even I can detect as I turn over his pages; its Huber, discovering through others' eyes what others could not discern with their own; its Goethe, its Haller, its Hunter, and mixed with practical greatness, its Pinel and its Jenner. Then there was Lavater, who showed how pure aesthetic estimation might be turned to the discovery of truth--a man depreciated because logicians and philodoxers can so much more easily detect his weakness than discern his strength. The nineteenth century, with its great thinker Darwin, its Pasteur (great in chemistry as well as in biology, a man who impressed me personally, and impresses me in his works as much as any but two or three of the century), its Lamaroll, Weissmann, Cuvier, Agassiz, von Baer, Bichat, Johannes Müller, Robert Brown, and I know not whom besides, has certainly garnered a magnificent harvest of great men from this field.

Those sciences which study individual objects and seek to explain them upon physical principles, astronomy, geology, etc. (corresponding to history and biography on the psychical side), demand the greatest assemblage of different powers. Those who pursue them have first to be mathematicians, physicists, chemists, naturalists, all at once, and, after that, astronomers or geologists, in addition. It is almost beyond human power. In the eighteenth century, A. G. Werner broke ground in geology, William Herschel, Kant, and Laplace did great things in astronomy. In the nineteenth century, geology was first really made a science, and among its great men one recalls at once Lyell, Agassiz, Kelvin. This country has

become its home. In astronomy, too, this country has been eminent, especially in the new astronomy which has afforded the needed scope for greatness, instead of the narrow rut that Bessel and Argelander had left behind them. Thus it happens that we have a magnificent group of great astronomers living among us to-day. We stand too close to them to take in their true proportions. But it is certain that the names of Chandler, Langley, Newcomb, Pickering, and several others are indelibly inscribed upon the heavens. In England it is only this year that Sir Norman Lockyer has brought the extraordinary research to which his life has been devoted to completion, so far as such work can be said to be capable of completion. It is an attribute of its greatness that it is endless.

When we compare all the men I have glanced at, with a view to eliciting a common trait somewhat distinctive of the nineteenth century, we cannot but see that science has been animated by a new spirit, till the very word has become a misnomer. It is the man of science, eager to have his every opinion regenerated, his every idea rationalized, by drinking at the fountain of fact, and devoting all the energies of his life to the cult of Truth, not as he understands it, but as he does not yet understand it, that ought properly to be called a philosopher. To an earlier age knowledge was power, merely that and nothing more; to us it is life and the summum bonum. Emancipation from the bonds of self, of one's own prepossessions, importunately sought at the hands of that rational power before which all must ultimately bow--this is the characteristic that distinguishes all the great figures of nineteenth-century science from those of former periods.