make any defence at all of this proceeding urge, in effect, that, however far the logician may push his criticisms of reasoning, still, in doing so, he must reason, and so must ultimately rely upon his instinctive recognition of good and bad reasoning. Whence it follows that, in Sigwart's words, 'every system of logic must rest upon this principle.' It is, however, to be noted that among the dicta of direct consciousness, many pronounce certain reasonings to be bad. If, therefore, such dicta are to be relied upon, man not only usually has a tendency to reason right, but also sometimes has a tendency to reason wrong; and if that be so, the validity of a reasoning cannot *consist* in a man's having a tendency to reason in that way. Some say that the validity of reasoning consists in the 'definitive dictum' of consciousness; but it has been replied that certain propositions in Euclid were studied for two thousand years by countless keen minds, all of whom had an immediate feeling of evidence concerning their proofs, until at last flaws were detected in those proofs, and are now admitted by all competent persons; and it is claimed that this illustrates how far from possible it is to make direct appeal to a definitive pronouncement. Besides, say those who object to this method, all reasoning and inquiry expects that there is such a thing as the truth concerning whatever question may be under examination. Now, it is of the very essence of this 'truth,' the meaning of the expectation, that the 'truth' in no wise depends upon what any man to whom direct appeal can be made may opine about that question. A *fortiori* it does not depend upon whether I am satisfied with it or not. It is further insisted that there can be no genuine criticism of a reasoning until that reasoning is actually doubted; and no sooner is it actually doubted than we find that consciousness has revoked her *dictum* in its favour, if she ever made any. It is, indeed, maintained that so far from true is it that every system of logic must be based upon any instinctive recognition of good and bad reasoning, that it is quite impossible for any reasoning to be based upon such recognition in respect to that same reasoning. In reasoning, a man may feel sure he is right; but to 'rest' that confidence on nothing but itself is to rest it on nothing at all. If the fact that we must use our reasoning instinct in criticizing reasoning proves that we must appeal to nothing else in such criticism, it equally proves that we ought to follow the lead of that in-

stinct without any logical control at all, which would be as much as to say that we ought not to reason at all. A man cannot criticize every part of his reasoning, since he cannot criticize the act of reasoning he is performing in the criticism, it is true. But he can criticize steps whose validity he doubts; and in doing so, ought to consider in what characters the validity of reasoning consists, and whether the reasoning in question possesses those characters.

Under an appeal to psychology is not meant every appeal to any fact relating to the mind. For it is, for logical purposes, important to discriminate between facts of that description which are supposed to be ascertained by the systematic study of the mind, and facts the knowledge of which altogether antecedes such study, and is not in the least affected by it; such as the fact that there is such a state of mind as doubt, and the fact that the mind struggles to escape from doubt. Even facts like these require to be carefully examined by the logician before he uses them as the basis of his doctrine. But many logicians have gone much further, and have avowedly based their systems upon one or another theory of psychology. Another class of logicians have professed to base logic upon a psychological theory of cognition. Of course, if this is done, such psychological doctrine is placed above logical criticism, or, at any rate, above logical support. For if the truth of a conclusion is known only from certain premises, it cannot be used to support those premises. Now, it may be doubted whether psychology is not, of all the special sciences, the one which stands most in need of appeal to a scientific logic.

Appeals to the usages of language are extremely common. They are made even by those who use algebraical notation in logic 'in order to free the mind from the trammels of speech' (Schröder, *Logik*, i. p. iii). It is difficult to see what can be hoped for from such a proceeding, unless it be to establish a psychological proposition valid for all minds. But to do this, it would be necessary to look beyond the small and very peculiar class of Aryan languages, to which the linguistic knowledge of most of those writers is confined. The Semitic languages, with which some of them are acquainted, are too similar to the Aryan greatly to enlarge their horizon. Moreover, even if other languages are examined, the value of any logical inferences from them is much diminished by the custom of our grammarians of violently fitting them to the Procrustean bed of Aryan grammar.

The objection which has been suggested to appeals to psychological results applies with far greater force to appeals to metaphysical philosophy, which, it will generally be conceded, can hardly take a step with security unless it rests upon the science of logic. Nevertheless, a great many logical treatises of various colours make it their boast that they are built upon philosophical principles.

Logicians occasionally appeal to the history of science. Such and such a mode of reasoning, it is said, for example, was characteristic of mediaevalism or of ancient science; such another produced the successes of modern science. If logic is to be based upon probable reasonings, as some logicians maintain that it must be, such arguments, if critically examined, must be admitted to have great weight. They will naturally be out of place in a system of logic which professes to demonstrate from certain initial assumptions that the kinds of reasoning it recommends must be accepted.

There is probably room for dispute as to whether logic need assert anything at all as an absolute matter of fact. If it does not, any appeal to experience would seem to be irrelevant. If it does, still the opinion may be that such assertions of logic are of so exceedingly broad and slight a nature that the universal experience of every man's every day and hour puts them beyond all doubt—such experiences as that the world presents appearances of variety, of law, and of the real action of one thing upon another. As appearances, these things do not seem likely ever to be doubted. If logic has need of any facts, and if such facts will suffice, no objection can well be made to an appeal to them.

The boundary between some parts of logic and pure mathematics in its modern treatment is almost evanescent, as may be seen in Dedekind's *Was sind und was sollen die Zahlen* (1888, Eng. trans. 1901). There are, however, departments of logic, such as the logic of probable inference (if that be regarded a part of logic), in which appeal is sometimes made to mathematical results, such as Bernoulli's law of high numbers. It seems to be the general opinion that nothing so difficult as mathematics can be admitted into, or be appealed to by, the science of logic, which has the peculiarity of consisting chiefly of truisms.

In mathematical reasoning there is a sort of observation. For a geometrical diagram or array of algebraical symbols is constructed according to an abstractly stated precept, and between the parts of such diagram or array

certain relations are observed to obtain, other than those which were expressed in the precept. These being abstractly stated, and being generalized, so as to apply to every diagram constructed according to the same precept, give the conclusion. Some logicians hold that an equally satisfactory method depends upon a kind of inward observation, which is not mathematical; since it is not diagrammatic, the development of a conception and its inevitable transformation being observed and generalized somewhat as in mathematics; and those logicians base their science upon such a method, which may conveniently be termed, and is sometimes termed, a Dialectic. Other logicians regard such a method as either extremely insecure or as altogether illusory.

The generally received opinion among professors of logic is that all the above methods may properly be used on occasion, the appeal to mathematics, however, being less generally recognized.

Literature: the history of logic in Western Europe, down to the revival of learning, is given by PRANTL, Gesch. d. Logik im Abendlande. Upon the points upon which this author touches, he always affords valuable information, though his judgments are peremptory and slashing. Unfortunately, he omits much which was regarded by the authors of whom he treats as most important, because he does not himself so regard it. He also omits much which would be interesting to a reader taking a broader conception of logic. It is hardly necessary to say that upon some large subjects his views are controverted. Of the modern development of logic there is no satisfactory history; but there are notices good as far as they go in UEBERWEG, Syst. d. Logik (Eng. trans.); in the much earlier work of BACHMANN, Syst. d. Logik (1828); in HAMILTON, Lects. on Logic; and for later work in B. ERDMANN, Logik. CH. SIGWART, Logic (Eng. trans.), and WUNDT, Logik, may also be profitably consulted. See under the logical topics generally (e. g. EMPIRICAL LOGIC, FORMAL LOGIC, JUDGMENT, and PROPOSITION); and also BIBLIOG, C.   (C.S.P., Ǝ.L.F.)

Logic (Hegel's *Logik*): see HEGEL'S TERMINOLOGY, II *b*.

Logic (exact): Ger. *exakte Logik*; Fr. *logique exacte*; Ital. *logica esatta*. The doctrine that the theory of validity and strength of reasoning ought to be made one of the 'exact sciences,' that is, that generalizations

from ordinary experience ought, at an early point in its exposition, to be stated in a form from which by mathematical, or expository, REASONING (q. v.), the rest of the theory can be strictly deduced; together with the attempt to carry this doctrine into practice.

This method was pursued, in the past, by Pascal (1623–62), Nicolas Bernoulli (1687–1759), Euler (1708–83), Ploucquet (1716–90), Lambert (1728–77), La Place (1749–1827), De Morgan (1806–71), Boole (1815–64), and many others; and a few men in different countries continue the study of the problems opened by the last two named logicians, as well as those of the proper foundations of the doctrine and of its application to inductive reasoning. The results of this method, thus far, have comprised the development of the theory of probabilities, the logic of relatives, advances in the theory of inductive reasoning (as it is claimed), the syllogism of transposed quantity, the theory of the Fermatian inference, considerable steps towards an analysis of the logic of continuity and towards a method of reasoning in topical geometry, contributions towards several branches of mathematics by applications of 'exact' logic, the logical graphs called after Euler and other systems for representing in intuitional form the relations of premises to conclusions, and other things of the same general nature.

There are those, not merely outside the ranks of exact logic, but even within it, who seem to suppose that the aim is to produce a calculus, or semi-mechanical method, for performing all reasoning, or all deductive inquiry; but there is no reason to suppose that such a project, which is much more consonant with the ideas of the opponents of exact logic than with those of its serious students, can ever be realized. The real aim is to find an indisputable theory of reasoning by means of the aid of mathematics. The first step in the order of logic towards this end (though not necessarily the first in the order of inquiry) is to formulate with mathematical precision, definiteness, and simplicity, the general facts of experience which logic has to take into account.

The employment of algebra in the investigation of logic is open to the danger of degenerating into idle trifling of too rudimentary a character to be of mathematical interest, and too superficial to be of logical interest. It is further open to the danger that the rules of the symbols employed may be mistaken for first principles of logic. An

algebra which brings along with it hundreds of purely formal theorems of no logical import whatever, must be admitted, even by the inventor of it, to be extremely defective in that respect, however convenient it may be for certain purposes. On the other hand, it is indisputable that algebra has an advantage over speech in forcing us to reason explicitly and definitely, if at all. In that way it may afford very considerable aid to analysis. It has been employed with great advantage in the analysis of mathematical reasonings.

Algebraic reasoning involves intuition just as much, as, though more insidiously than, does geometrical reasoning; and for the investigation of logic it is questionable whether the method of graphs is not superior. Graphs cannot, it is true, readily be applied to cases of great complexity; but for that very reason they are less liable to serve the purposes of the logical trifler. In the opinion of some exact logicians, they lead more directly to the ultimate analysis of logical problems than any algebra yet devised. See LOGICAL DIAGRAM (OR GRAPH).

It is logical algebra, however, which has chiefly been pursued. De Morgan invented a system of symbols, which had the signal advantage of being entirely new and free from all associations, misleading or otherwise. Although he employed them for synthetical purposes almost exclusively, yet the great generality of some of the conceptions to which they led him is sufficient to show that they might have been applied with great advantage in analysis. Boole was led, no doubt from the consideration of the principles of the calculus of probabilities, to a wonderful application of ordinary algebra to the treatment of all deductive reasoning not turning upon any relations other than the logical relations between non-relative terms. By means of this simple calculus, he took some great steps towards the elucidation of probable reasoning; and had it not been that, in his pre-Darwinian day, the notion that certain subjects were profoundly mysterious, so that it was hopeless, if not impious, to seek to penetrate them, was still prevalent in Great Britain, his instrument and his intellectual force were adequate to carrying him further than he actually went. Most of the exact logicians of to-day are, from the nature of the case, followers of Boole. They have modified his algebra by disusing his addition, subtraction, and division, and by introducing a sign of logical aggregation. This was first

done by Jevons; and he proposed ⫿, a sign of division turned up, to signify this operation. Inasmuch as this might easily be read as three signs, it would perhaps be better to join the two dots by a light curve, thus Ψ. Some use the sign + for logical aggregation. The algebra of Boole has also been amplified so as to fit it for the logic of relatives. The system is, however, far from being perfect. See RELATIVES (logic of).

Certain terms of exact logic may be defined as follows :—

Aggregation. The operation of uniting two or more terms or propositions, called *aggregants*, to produce an *aggregate* term or proposition which is true of everything of which any aggregant is true, and false of everything of which all the aggregants are false. It is opposed to *composition*, which is the operation of producing from two or more terms or propositions, called the *components*, a new term or proposition, called their *compound*, which is true of all of which all the components are true, and false of all of which any are false.

Absorption, law of (Ger. *Absorptionsgesetz*). The proposition that if of two aggregants one contains the other as a component, the aggregate is identical with the latter.

Alternative proposition. A term preferred by some logicians to 'disjunctive,' because the latter term is often, as by Cicero and Aulus Gellius, understood to imply that one, and one only, of the alternatives is true. At the same time, the standard traditional example of a disjunctive was 'Socrates currit vel Plato disputat,' and the rule was 'Ad veritatem disiunctivae sufficit alteram partem esse veram.' Nevertheless, the narrower sense was also recognized, and the term alternative is perhaps preferable.

Associative. An operation combining two elements is *associative* if, and only if, in combining the result with a third element, it makes no difference whether the middle element be first combined with the last and the result with the first, or the other way, so long as the order of sequence is preserved. Addition and multiplication are associative, while involution is not so; for ten to the three-square power is a milliard, while ten cube squared is only a million. An associative algebra is an algebra in which multiplication is associative.

Commutative. An operation by which two elements are united is said to be *commutative* if, and only if, it makes no difference which is

taken first. Thus, because twice three is thrice two, numerical multiplication is commutative.

Composition: see *Aggregation*, above.

Compound: see *Aggregation*, above.

Copula is often defined as that which expresses the relation between the subject-term and the predicate-term of a proposition. But this is not sufficiently accurate for the purposes of exact logic. Passing over the objection that it applies only to categorical propositions, as if conditional and copulative propositions had no copula, contrary to logical tradition, it may be admitted that a copula often does fulfil the function mentioned; but it is only an accidental one, and its essential function is quite different. Thus, the proposition 'Some favoured patriarch is translated' is essentially the same as 'A translated favoured patriarch is'; and 'Every mother is a lover of that of which she is a mother' is the same as 'A mother of something not loved by her is not.' In the second and fourth forms, the copula connects no terms; but if it is dropped, we have a mere term instead of a proposition. Thus the essential office of the copula is to express a relation of a general term or terms to the universe. The universe must be well known and mutually known to be known and agreed to exist, in some sense, between speaker and hearer, between the mind as appealing to its own further consideration and the mind as so appealed to, or there can be no communication, or 'common ground,' at all. The universe is, thus, not a mere concept, but is the most real of experiences. Hence, to put a concept into relation to it, and into the relation of describing it, is to use a most peculiar sort of sign or thought; for such a relation must, if it subsist, *exist* quite otherwise than a relation between mere concepts. This, then, is what the copula essentially does. This it may do in three ways: first, by a vague reference to the universe collectively; second, by a reference to all the individuals existent in the universe distributively; third, by a vague reference to an individual of the universe selectively. 'It is broad daylight,' I exclaim, as I awake. My universe is the momentary experience as a whole. It is that which I connect as object of the composite photograph of daylight produced in my mind by all my similar experiences. Secondly, 'Every woman loves something' is a description of every existing individual in the universe. Every such individual is said to be coexistent only with what, so far as it

is a woman at all, is sure to be a lover of some existing individual. Thirdly, 'Some favoured patriarch is translated' means that a certain description applies to a select individual. A hypothetical proposition, whether it be conditional (of which the alternative, or disjunctive, proposition is a mere species, or *vice versa*, as we choose to take it) or copulative, is either general or *ut nunc*. A general conditional is precisely equivalent to a universal categorical. 'If you really want to be good, you can be,' means 'Whatever determinate state of things may be admissibly supposed in which you want to be good is a state of things in which you can be good.' The universe is that of determinate states of things that are admissible hypothetically. It is true that some logicians appear to dispute this; but it is manifestly indisputable. Those logicians belong to two classes: those who think that logic ought to take account of the difference between one kind of universe and another (in which case, several other *substantiae* of propositions must be admitted); and those who hold that logic should distinguish between propositions which are necessarily true or false together, but which regard the fact from different aspects. The exact logician holds it to be, in itself, a defect in a logical system of expression, to afford different ways of expressing the same state of facts; although this defect may be less important than a definite advantage gained by it. The copulative proposition is in a similar way equivalent to a particular categorical. Thus, to say 'The man might not be able voluntarily to act otherwise than physical causes make him act, whether he try or not,' is the same as to say that there is a state of things hypothetically admissible in which a man tries to act one way and voluntarily acts another way in consequence of physical causes. As to hypotheticals *ut nunc*, they refer to no range of possibility, but simply to what is true, vaguely taken collectively.

Although it is thus plain that the action of the copula in relating the subject-term to the predicate-term is a secondary one, it is nevertheless necessary to distinguish between copulas which establish different relations between these terms. Whatever the relation is, it must remain the same in all propositional forms, because its nature is not expressed in the proposition, but is a matter of established convention. With that proviso, the copula may imply any relation whatsoever. So understood, it is the *abstract copula* of De Morgan (*Camb. Philos. Trans.*, x. 339). A *transitive* copula is one for which the mood Barbara is valid. Schröder has demonstrated the remarkable theorem that if we use is in small capitals to represent any one such copula, of which 'greater than' is an example, then there is some relative term *r*, such that the proposition '*S* is *P*' is precisely equivalent to '*S* is *r* to *P* and is *r* to whatever *P* is *r* to.' A *copula of correlative inclusion* is one for which both Barbara and the formula of identity hold good. Representing any one such copula by *is* in italics, there is a relative term *r*, such that the proposition '*S* is *P*' is precisely equivalent to '*S* is *r* to whatever *P* is *r* to.' If the last proposition follows from the last but one, no matter what relative *r* may be, the copula is called the *copula of inclusion*, used by C. S. Peirce, Schröder, and others. De Morgan uses a copula defined as standing for any relation both transitive and convertible. The latter character consists in this, that whatever terms *I* and *J* may be, if we represent this copula by is in black=letter, then from '*I* is *J*' it follows that '*J* is *I*.' From these two propositions, we conclude, by Barbara, that '*I* is *I*.' Such copulas are, for example, 'equal to,' and 'of the same colour as.' For any such copula there will be some relative term *r*, such that the proposition '*S* is *P*' will be precisely equivalent to '*S* is *r* to everything, and only to everything, to which *P* is *r*.' Such a copula may be called a copula of correlative identity. If the last proposition follows from the last but one, no matter what relative *r* may be, the copula is the *copula of identity* used by Thomson, Hamilton, Baynes, Jevons, and many others.

It has been demonstrated by Peirce that the copula of inclusion is logically simpler than that of identity.

Diagram: see LOGICAL DIAGRAM.

Dialogism. A form of reasoning in which from a single premise a disjunctive, or alternative, proposition is concluded introducing an additional term; opposed to a syllogism, in which from a copulative proposition a proposition is inferred from which a term is eliminated.

Syllogism.

All men are animals, and all animals are mortal ;

∴ All men are mortal.

Dialogism.

Some men are not mortal ;

∴ Either some men are not animals, or some animals are not mortal.

Dimension. An element or respect of extension of a logical universe of such a nature that the same term which is individual in one such element of extension is not so in another. Thus, we may consider different persons as individual in one respect, while they may be divisible in respect to time, and in respect to different admissible hypothetical states of things, &c. This is to be widely distinguished from different universes, as, for example, of things and of characters, where any given individual belonging to one cannot belong to another. The conception of a multidimensional logical universe is one of the fecund conceptions which exact logic owes to O. H. Mitchell. Schröder, in his then second volume, where he is far below himself in many respects, pronounces this conception 'untenable.' But a doctrine which has, as a matter of fact, been held by Mitchell, Peirce, and others, on apparently cogent grounds, without meeting any attempt at refutation in about twenty years, may be regarded as being, for the present, at any rate, tenable enough to be held.

Dyadic relation. A fact relating to two individuals. Thus, the fact that *A* is similar to *B*, and the fact that *A* is a lover of *B*, and the fact that *A* and *B* are both men, are dyadic relations; while the fact that *A* gives *B* to *C* is a triadic relation. Every relation of one order of relativity may be regarded as a relative of another order of relativity if desired. Thus, *man* may be regarded as *man coexistent with*, and so as a relative expressing a dyadic relation, although for most purposes it will be regarded as a monad or non-relative term.

Index (in exact logic): see sub verbo.

Many other technical terms are to be found in the literature of exact logic.

Literature: for the study of exact logic in its more recent development, excluding probability, the one quite indispensable book is SCHRÖDER, *Algebra d. Logik*; and the bibliography therein contained is so exhaustive that it is unnecessary to mention here any publications previous to 1890. Schröder's pains to give credit in full measure, pressed down and running over, to every other student is hardly less remarkable than the system, completeness, and mathematical power of his work, which has been reviewed by C. S. PEIRCE in the Monist, vii. 19–40, 171–217. See also C. S. PEIRCE, Studies in Logic ; Pop. Sci. Mo., xii. 1 ; and Proc. Amer. Acad. Arts and Sci., vii. 287. Cf. SCIENTIFIC METHOD.
(C.S.P.)

Logic (of chance): see PROBABILITY.

Logic (of emotion): see TERMINOLOGY, English, 'Affective Logic.'

Logic (social): see SOCIAL LOGIC.

Logic (symbolic): see SYMBOLIC LOGIC.

Logical [Lat. *logicalis*, from *logica*, logic] : Ger. *logisch* ; Fr. *logique* ; Ital. *logico*. Irrespective of any facts except those of which logic needs to take cognizance, such as the facts of doubt, truth, falsity, &c.

Logical possibility is, according to usage, freedom from all contradiction, explicit or implicit; and any attempt to reform the inaccuracy would only bring confusion.

Logical necessity is the necessity of that whose contrary is not logically possible.

Logical induction is an induction based on examination of every individual of the class to which the examination relates. Thus, conclusions from a census are logical inductions. While this mode of inference is a degenerate form of induction, it also comes into the class of dilemmatic reasoning.

Logical truth is a phrase used in three senses, rendering it almost useless.

1. The harmony of a thought with itself. Most usually so defined, but seldom so employed. So far as this definition is distinct, it makes logical truth a synonym for logical possibility; but, no doubt, more is intended (Hamilton, *Lects. on Logic*, xxvii).

2. The conformity of a thought to the laws of logic; in particular, in a concept, consistency; in an inference, validity; in a proposition, agreement with assumptions. This would better be called *mathematical truth*, since mathematics is the only science which aims at nothing more (Kant, *Krit. d. reinen Vernunft*, 1st ed., 294).

3. More properly, the conformity of a proposition with the reality, so far as the proposition asserts anything about the reality. Opposed, on the one hand, to metaphysical truth, which is an affection of the *ens*, and on the other hand to ethical truth, which is telling what a witness believes to be true (Burgersdicius, *Inst. Met.*, chap. xviii).

Logical parts and whole. Parts and whole of logical extension.

Logical reasoning. Reasoning in accordance with a LEADING PRINCIPLE (q. v.) which thorough analysis, discussion, and experience have shown must lead to the truth, in so far as it is relied upon. But what Aristotle