Einleitung in die Philos., 63–90; KÜLPE, Einleitung in die Philos., 127–37 (who distinguishes the different logical forms of materialisms). See also PAUL JANET, Le Matér. contemporain (6th ed.). Recent materialistic views in psychology are expounded by BALDWIN, Recent Discussion in Materialism, Presb. and Ref. Rev., i. (1890) 357. Cf. also the Encyclopedias, sub verbo; and BIBLIOG. B, 2, *e*. (A.S.P.P.)

Mathematical Economics: Ger. *mathematische Oekonomik*; Fr. *économie mathématique*; Ital. *economia matematica*. The development of notations and functions which serve to connect our experience with regard to balance of motives with our observation as to quantities of wealth; thus enabling the study of either set of phenomena to be used to explain the others quantitatively.

The differential equations of economics are mostly in terms of motive, the integrals mostly in terms of wealth.

The pioneer in mathematical economics was Cournot; his work has been extended by Dupuit and by Marshall. The efforts of v. Thünen were less permanently fruitful than those of Cournot. See SUPPLY AND DEMAND.

Another line of thought, first developed by Gossen, and afterwards (non-mathematically) by Menger, was made an instrument of powerful analysis by Jevons, and by the Austrian school after him (see MARGINAL INCREMENT). Among more recent mathematical economists we may note Auspitz and Lieben, Edgeworth, Fisher, and Pareto.

As Cohn well observes, mathematical methods in economics must not be confounded with statistical ones. The latter represent the extreme of concreteness, the former the extreme of abstraction. (A.T.H.)

Mathematical Logic: Ger. (1) *Logik der Mathematik*; Fr. (1) *logique des mathématiques*; Ital. (1) *logica della matematica*. (1) The logical analysis of mathematics. (C.S.P.)

(2) SYMBOLIC LOGIC (q. v.).

Literature (to 1): the logic of arithmetic is treated by DEDEKIND in his Was sind und was sollen die Zahlen? (Eng. trans. in Essays on Number, 1901). See also the ninth lecture of the third volume of SCHRÖDER, Logik; and FINE, Number System of Algebra. For the logic of the calculus, see the second edition of JORDAN, Cours d'Analyse; also CLIFFORD, Theory of Metrics, in his Mathematical Papers; WEBER, Algebra; and the papers of G. CANTOR, some of which are contained in the Acta Mathematica, ii, and subsequent ones in the Mathematische Annalen, 15, 17, 20, 21, 23, 46, 49. LISTING's papers on topical geometry are two, one in the Gött. Abhand., the other in the Gött. Nachr. Several of RIEMANN's papers are valuable in a logical point of view. See also CAUCHY, Théorie des Clefs. PETERSEN, Methods and Theories, shows how to solve problems in elementary geometry. Cf. MATHEMATICS. (C.S.P.)

Mathematics [Gr. μαθηματική, from μαθήματα, things learned]: Ger. *Mathematik*; Fr. *mathématiques*; Ital. *matematica*. A science of abstract relationships. These are by no means exclusively quantitative. The projective properties of curves, for instance, of which an example is given below, are purely positional. And the whole of analysis may be presented in the form of an abstract calculus of symbols, to which no meaning of any kind need be assigned, the operations themselves being defined by certain formal laws, as addition by the laws $a+b=b+a$ and $(a+b)+c=a+(b+c)$. (J.M.B.–H.B.F.)

'Any conception which is definitely and completely determined by means of a finite number of specifications, say by assigning a finite number of elements, is a mathematical conception. Mathematics has for its function to develop the consequences involved in the definitions of a group of mathematical conceptions' (Chrystal) combined with certain fundamental principles, or axioms and postulates. (C.L.F., F.F.)

One of the most distinctive characteristics of mathematics is the extreme definiteness of the conceptions with which it deals. They admit of exact definition by a limited number of marks.

The more fundamental of these conceptions correspond immediately to things and relations among things in the external world, from which, in fact, they have been derived by a process of abstraction. Such are the conceptions of cardinal numbers and of the ordinal arrangement of the cardinal numbers on which arithmetic is based (see NUMBER); the conceptions of point, line, &c., and of such fundamental relations as that two points determine a right line, &c., which lie at the basis of geometry.

It is the function of mathematics to make the simplest possible selection of such of these primary conceptions as are mutually independent, and by combining them and generalizing them, to create a body of more complex conceptions which have intrinsic interest and beauty and a value for the furtherance of the science itself or for the study of other