mands? Cf. One (the), and Unity and Plurality.

The Eleatic philosophy pronounced the appearance of multiplicity to be an illusion of the senses; and Zeno, by a series of indirect arguments, endeavours to demonstrate its impossibility. The same problem (ἓν τὰ πολλὰ εἶναι καὶ τὸ ἓν πολλὰ) reappears in Plato, who reduces the multiplicity of sense-phenomena to the unity of the idea in which they participate or which they represent. The multiplicity of the sense-world appears to be regarded here also as a species of illusion. But Plato recognizes a multiplicity within the ideal world itself, in virtue of what has been called the community of concepts (κοινωνία τῶν γένων), or the participation of the ideas in one another. This world of ideas thus differs from the abstract unity of the Eleatics in being rather a series of ideas which dialectically imply one another. As he says in the *Philebus* (15 D): 'The One and the Many run about everywhere together, in and out of every word which is uttered, as they have done in all time past as well as present; and this union of them will never cease, and is not now beginning, but is, I believe, an everlasting quality of thought itself, which never grows old in us.'

The same question of the One and the Many is the underlying motive of the scholastic disputes between nominalism and realism, and gives a pantheistic or an individualistic bias to the systems of most philosophers.

In the Kantian philosophy, the contribution of sense to knowledge is spoken of as a mere Manifold (*Mannigfaltiges*), a multiplicity or diversity of particulars. The synthetic function of the understanding must supervene with its categories or connective notions upon these passively apprehended units of sense before we can speak of knowledge or experience. (A.S.P.P.)

Multitude (in mathematics) [Lat. *multitudo*]: Ger. *Mächtigkeit, Cardinalzahl*; Fr. *puissance*; Ital. *moltitudine*. That relative character of a collection which makes it greater than some collections and less than others. A collection, say that of the *A's*, is *greater than* another, say that of the *B's*, if, and only if, it is impossible that there should be any relation r, such that every A stands in the relation r to a B to which no other A is in the relation r.

The precise analysis of the notion is due to G. Cantor, whose definition is, however, a little different in its mode of expression, since it is more abstract. He defines the character in these words: 'By Mächtigkeit or cardinal number of a collection (Menge) M, we mean the universal concept, which by the help of our active faculty of thought results from the collection M by abstraction from the characters of the different members (Elemente) of that collection and from the order in which they are given (Gegebensein).

A cardinal number, though confounded with multitude by Cantor, is in fact one of a series of vocables the prime purpose of which, quite unlike any other words, is to serve as an instrument in the performance of the experiment of counting; these numbers being pronounced in their order from the beginning, one as each member of the collection is disposed of in the operation of counting. If the operation comes to an end by the exhaustion of the collection, the last cardinal number pronounced is applied adjectivally to the collection, and expresses its multitude, by virtue of the theorem that a collection the counting of which comes to an end, always comes to an end with the pronunciation of the same cardinal number.

If the cardinal numbers are considered abstractedly from their use in counting, simply in themselves, as objects of mathematical reasoning, stripped of all accidents not pertinent to such study, they become indistinguishable from the similarly treated ordinal numbers, and are then usually called *ordinal numbers* by the mathematico-logicians. There is small objection to this; yet it is to be remarked that they are ordinal in different senses in grammar and in the logic of mathematics. For in grammar they are called ordinal as being adapted to express the ordinal places of other things in the series to which those things belong; while in the logic of mathematics the only relevant sense in which they are ordinal is as being defined by a serial order within their own system. The definition of this order is not difficult; but the syntax of ordinary language does not lend itself to the clear expression of such relations in the manner in which they ought to be expressed in order to bring out their logical character. It must, therefore, be here passed by. In fact, none of the doctrines of logic can be satisfactorily expressed under the limitations here imposed, however simple they may be. The doctrine of ordinal numbers is by Dedekind (*Was sind und was sollen die Zahlen?*) made to precede that of the cardinal numbers; and this is logically preferable, if hardly so imperative as Schröder considers it.

The doctrine of the so-called ordinal numbers is a doctrine of pure mathematics; the doctrine of cardinal numbers, or, rather, of multitude, is a doctrine of mathematics applied to logic. The smallest multitude is most conveniently considered to be *zero*; but this is a question of definition. A *finite* collection is one of which the syllogism of transposed quantity holds good. Of finite collections, it is true that the whole is greater than any part. It is singular that this is often taken as the type of an axiom, although it has from early times been a matter of familiar knowledge that it is not true of infinite collections. Every addition of one increases a finite multitude. An infinite collection cannot be separated into a lesser collection of parts all smaller than itself. The multitude of all the different finite multitudes is the smallest infinite multitude. It is called the *denumeral* multitude. (Cantor uses a word equivalent to *denumerable*; but the other form has the advantage of being differentiated from words like *enumerable, abnumerable,* which denote classes of multitudes, not, like *denumeral*, a single multitude.) Following upon this is a denumeral series of multitudes called by C. S. Peirce the *first, second,* &c. *abnumerable* multitudes. Each is the multitude of possible collections formed from the members of a collection of the next preceding multitude. They seem to be the same multitudes that are denoted by Cantor as *Alephs*. The first of them is the multitude of different limits of possible convergent series of rational fractions, and therefore of all the quantities with which mathematical analysis can deal under the limitations of the doctrine of limits. (The imaginaries do not increase the multitude.) What comes after these is still a matter of dispute, and is perhaps of inferior interest. The transition to continuity is, however, a matter of supreme importance for the theory of scientific method; nor is it a very complicated matter; but it cannot be stated under the limitations of expression here imposed upon us. (C.S.P., H.B.F.)

Literature: see NUMBER.

Mundane: see MUNDUS.

Mundus [Lat.]: Ger. *Welt*; Fr. *monde*; Ital. *mondo*. The term used by the Romans to render the Greek κόσμος, the visible orderly system of the world, with more particular reference to the heavens and the heavenly bodies, whose regular motions first impressed the idea of order on primitive thought.

Cicero's definition (*Tim.* 10) retains this reference: 'ut hunc hac varietate distinctum bene Graeci κόσμον, nos lucentem mundum nominaremus.' In so far as this system is contrasted with a preceding state of things—whether chaos or primitive elements—the κόσμος or *mundus* is regarded as limited both in time and in space, and is not therefore to be identified with the universe (τὸ πᾶν, *omne*). The Epicurean philosophy in particular supposes innumerable worlds (in some respects perhaps resembling, in many more probably differing from, the world-system we know) to result from the mechanical clashings of the atoms in infinite space throughout infinite time. Each world-system is girdled from the embrace of hungry space by an outer envelope of fire or ether—the 'flammantia moenia mundi' of Lucretius' account. In the 'intermundia' or intermundane spaces Epicurus supposed the gods to reside. Cf. Lucretius, *De rerum Natura*, iii. 16–22, finely rendered by Tennyson in his poem *Lucretius*.

The terms *mundus sensibilis* and *mundus intelligibilis* were used to express the Platonic contrast between the world of sense-perception, which is a world of phenomena or mere appearance, and the ideal world, the world of noumena or of ultimate reality. They were appropriated by Kant, in a somewhat altered sense, to denote the world of nature or of categorized sensation, to which he limits our knowledge; and the intelligible world (Verstandeswelt), which is for the theoretic reason a merely negative or limitative conception, but which the practical reason reveals as a realm of ethical ends and moral freedom. It is in connection with this Kantian distinction that the term mostly occurs in modern philosophical writing. (A.S.P.P.)

Mundane and extra-mundane are used respectively for what is and what is not subject to the conditions of the physical world. (H.R.S.)

Muscae Volitantes [Lat. *musca*, a fly, + *volitans*, dancing]: Ger. *fliegende Mücken*; Fr. *mouches volantes*; Ital. *mosche volanti*. Variable entoptic appearances, due to the presence of small foreign bodies in the vitreous humour. They take the form of bright worm-like threads, strings of glistening beads, groups of bright dots, tiny circles with brighter centres, &c., and usually travel downward in the field of vision (i. e. upward in the humour). Cf. ENTOPTIC PHENOMENA.

Literature: HELMHOLTZ, Physiol. Optik

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