experience. But the conditions operative at the time of reproduction may be such as to make unmodified reinstatement impossible. They may be such that if the parts are revived without alteration, their relation cannot be reinstated; or inversely, if the relation is reinstated, the related presentations must be modified. It may happen, and very commonly does happen, that the presentation which starts the reproductive process is not a mere repetition of the corresponding constituent of the original whole. It may vary considerably from this original constituent without losing its redintegrative tendency. Suppose the original combination to be *ab*, where *a* stands to *b* in a certain relation *r*. Suppose *a* to recur in the modified form *a*. Inasmuch as *a* partakes of the nature of *a*, it will tend as far as may be to reconstitute the whole *ab*. But the difference between *a* and *a* may be such as to require a corresponding differentiation of *b* if the relation *r* is to be reinstated. It may happen that *b* cannot assume the same relation to *a* as that in which it stood to *a*. Thus the revival of *b* will not be a reconstitution of the original whole, because the relation *r* has disappeared. On the other hand, if the relation *r* is recalled, the other term of the relation must be modified. For *b* there must be substituted *β*, which is related to *a* as *b* was related to *a*. This is relative suggestion.

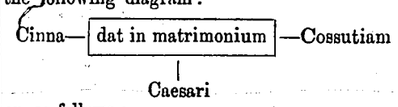
What actually takes place on any given occasion depends on the special conditions operative at the time. The more fully and intensely we are interested in the whole as such, the stronger will be the tendency to revive the relation *r* and to modify *b* so as to transform it into *β*. This tendency may take effect at once, so that *a* immediately calls up *β* without the previous intervention of other mental processes. Thus in the very act of making a pun or a rhyme we may alter the pronunciation of a word. If we are looking for a place on a map which we know by actual travelling to be a certain distance from London, we allow for the difference in scale between the actual distance and that on the map without express comparison or explicit formulation. The imitative plays of children are full of such mental accommodations, the doll, for instance, being treated as a baby *mutatis mutandis*.

In such cases relative suggestion operates or may operate immediately, in others it does not take effect without an intervening mental operation involving express comparison between the present case and its analogue. When this happens the process is one of reasoning in the proper sense. Suppose we have to throw a piece of paper upwards for a considerable distance. We are used to throwing stones as far or further; but a piece of paper is different. If the problem presented itself for the first time, it is very unlikely that any one but a genius would proceed immediately to wrap the paper round a pebble and then throw. There would be a previous comparison of the case of throwing a stone and throwing a piece of paper, and the relative suggestion would be brought to birth by the help of this preliminary process.

It will be seen that relative suggestion depends on the interest which a certain kind of relation or form of combination possesses. It would be possible to classify different types of mind from this point of view. In some minds rhetorical antithesis, in others metaphor, in others syllogistic form, in others a triple dialectic movement preponderates and determines relative suggestion. (G.F.S.)

Relatives (logic of): Ger. *logische Beziehungslehre*; Fr. *logique des relations* (L.C.); Ital. *termini relativi* (relative terms). If from any proposition having more than one subject (used to include 'objects') we strike out the indices of the subjects, as in '—— praises —— to ——,' '—— dat in matrimonium ——,' what remains and requires at least two insertions of subject-nouns to make a proposition is a 'relative term,' or '*relative rhema*,' called briefly a 'relative.' The relative may be converted into a complete assertion by filling up the blanks with proper names or abstract nouns; this serves as a criterion.

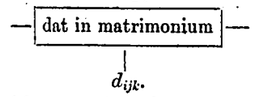
But in such a relative there must be such an idea of the difference between the subjects to be applied that 'dat in matrimonium' shall be different from 'datur in matrimonium.' In order to free ourselves from the accidents of speech, we might represent the sentence by the following diagram:



or, as follows:

d_{ijk} (Cinna $= i$, Cossutia $= j$, Caesar $= k$).

Then the relative will appear as



or as d_{ijk}.

But, in either case, in order to explain what

is meant, it will be necessary to explain how those three tails, or the three letters *i*, *j*, *k*, differ. The order shows which of three indices is given, which giver, which recipient.

Relatives may be more or less general like other terms, that is, one relative may be predicable of members of a set of which another is not, while the latter is predicable only of members of sets of which the former is predicable. By a set is meant an ordered system, so that *ABC* and *BCA*, though the same collection, are different sets. As any general term is predicable of any one of an aggregate of individuals, so a relative is predicable of any one of an aggregate of sets; and each such set may be regarded as an individual relative. By a system is meant an individual of which if anything is true, the truth of it consists in certain things being true of certain other individuals, called its *members*, regardless of the system. A *system* is either a *sorite*, *heap*, or *mere collection*, or it is a *set*. A *sorite* is a system of which, if anything is true, its truth consists of the truth of one predicate for any one of the members. A *set* is a system of which the truth of anything consists in the truth of different predicates. Of course the idea of relation is involved in the idea of a system. As it is very important for the understanding of relations that the conception of a system should be perfectly clear, let us consider the latter a moment in its simplest form, that of a *sorite* or mere collection. *ABC* is a sorite. Thus, it is true of it that it contains the three first letters of the alphabet, and the truth of that consists in *A*, *B*, and *C* being each one of the first three letters of the alphabet. It is true that it contains nothing but the first letters of the alphabet, because it is true of *A*, *B*, *C* severally that each is nothing but one of the first three letters of the alphabet. *AB* is a different sorite, because something is true of it which is not true of *ABC*. *A* may be regarded as a sorite provided we mean not *A* in its first intention and being, but a something whose being consists in *A*'s being. The collection *A* is not the letter *A*, but it contains *A* and nothing else. If it be said that there is no such thing, the reply is that every collection, every system may be said to be an *ens rationis*. To this point we shall return. Even Nothing may be said to be a collection. For when we say that Nothing is less than 1, we do not mean that a self-subsisting individual is so, but that an *ens rationis* whose mode of being consists in the absence of everything is less than 1. The sorite *ABC* is other than *ABΓ*. But should I say that *ABC* contains two of the letters of Caesar's first name, and subsequently learn that that was a mistake, the real name being Gaius, that would not make *ABC* a different sorite.

That in the reality which corresponds to a proposition with a relative predicate is called the *fundamentum relationis*. A *relationship* is a system of such fundamenta. *Relation* is the relative character, conceived as belonging in different ways to the different relates, and (owing to the somewhat undue prominence given by familiar languages to one of these) especially to the relate which is denoted by the noun which is the subject nominative.

Relatives and relations are said to differ in their *orders*, according to the numbers of their relates. *Dyadic* or *dual* relations, or relatives of two relates, of which the second is called the *correlate*, differ somewhat widely from *plural*, or *polyadic*, relations. *Triadic* relations have all the principal characters of *tetradic* and higher relations. In fact, a compound of two triadic relatives may be a tetradic relative; as 'praiser of —— to a maligner of —— to ——'.

Relatives may be compounded in all the ways in which other terms can be compounded as well as in other ways closely related to those. Thus, *A* may be said to be at once a lover and a servant of *B*, and it may be said that there is something, *X*, such that *A* is a lover of *X*, while *X* is a servant of *B*; so that *A* is a lover of a servant of *B*. This mode of composition is called *relative multiplication*. So, not only may it be said that *A* is either a lover or a servant of *B* (not excluding both), but also that whatever *X* may be, either *A* is a lover of *X* or *X* is a servant of *B*; that is, *A* is a lover of everything there is besides servants of *B*. (This wording, by Schröder, slightly violates English idiom, but is valuable as showing the analogy to aggregation.) This mode of composition is called *relative addition*. So, again, it may not only be said that *A* is if a lover then a servant of *B*, but also that whatever *X* may be, if *A* is a lover of *X*, then *X* is a servant of *B*; that is, *A* is a lover only of servants of *B*. This is called *relative regressive involution*. Or it may be said that whatever *X* may be, *A* is a lover of *X*, if *X* is a servant of *B*, or *A* is a lover of whatever is a servant of *B*. This

is called *relative progressive involution*. Polyadic relatives are capable of other modes of composition. Thus, it may be said that anything whatever, *X*, being taken, something *Y* exists, such that *A* praises *X* to *Y* while *X* maligns *Y* to *B*; that is, *A* praises everybody to somebody maligned by him to *B*. Or we can say that there is something *Y*, such that, whatever *X* may be, *A* praises *X* to *Y* while *X* maligns *Y* to *B*; or, *A* praises everybody to somebody whom everybody maligns to *B*.

Deductive logic can really not be understood without the study of the logic of relatives, which corrects innumerable serious errors into which not merely logicians, but people who never opened a logic-book, fall from confining their attention to non-relative logic. One such error is that demonstrative reasoning is something altogether unlike observation. But the intricate forms of inference of relative logic call for such studied scrutiny of the representations of the facts, which representations are of an *iconic* kind, in that they represent relations in the fact by analogous relations in the representation, that we cannot fail to remark that it is by *observation* of diagrams that the reasoning proceeds in such cases. We successively simplify them and are always able to remark that such observation is required, and that it is even thus, and no otherwise, that the conclusion of a simple syllogism is seen to follow from its premises. Again, non-relative logic has given logicians the idea that deductive inference was a following out of a rigid rule, so that machines have been constructed to draw conclusions. But this conception is not borne out by relative logic. People commonly talk of *the* conclusion from a pair of premises, as if there were but one inference to be drawn. But relative logic shows that from any proposition whatever, without a second, an endless series of necessary consequences can be deduced; and it very frequently happens that a number of distinct lines of inference may be taken, none leading into another. That this must be the case is indeed evident without going into the logic of relatives, from the vast multitude of theorems deducible from the few incomplex premises of the theory of numbers. But ordinary logic has nothing but a barren sorites to explain how this can be. Since Kant, especially, it has been customary to say that deduction only elicits what was implicitly thought in the premises; and the famous distinction of analytical and synthetical judgments is based upon that notion. But the logic of relatives shows that this is not the case in any other sense than one which reduces it to an empty form of words. Matter entirely foreign to the premises may appear in the conclusion. Moreover, so far is it from being true, as Kant would have it, that all reasoning is reasoning in *Barbara*, that that inference itself is discovered by the microscope of relatives to be resolvable into more than half a dozen distinct steps. In minor points the doctrines of ordinary logic are so constantly modified or reversed that it is no exaggeration to say that deductive logic is completely metamorphosed by the study of relatives.

One branch of deductive logic, of which from the nature of things ordinary logic could give no satisfactory account, relates to the vitally important matter of abstraction. Indeed, the student of ordinary logic naturally regards abstraction, or the passage from 'the rose smells sweet' to 'the rose has perfume,' to be a quasi-grammatical matter, calling for little or no notice from the logician. The fact is, however, that almost every great step in mathematical reasoning derives its importance from the fact that it involves an abstraction. For by means of abstraction the transitory elements of thought, the *ἔπεα πτερόεντα*, are made substantive elements, as James terms them, *ἔπεα ἀπτερόεντα*. It thus becomes possible to study their relations and to apply to these relations discoveries already made respecting analogous relations. In this way, for example, operations become themselves the subjects of operations.

To take a most elementary example—from the idea of a particle moving, we pass to the idea of a particle describing a line. This line is then thought as moving, and so as generating a surface; and so the relations of surfaces become the subject of thought. An abstraction is an *ens rationis* whose being consists in the truth of an ordinary predication. A *collection*, or *system*, is an abstraction or abstract *ens*; and thus the whole doctrine of number is founded on the operation of abstraction. If we conceive an object to be a collective whole, but to be so in such a way that it has no part which is not itself a collective whole in the same way, then, if the collection is of the nature of a sorite, it is a *genera*, whose parts are distinguished merely as having additional characters; but if the collection is a *set*, whose members have other relations to one another, it is a *continuum*.

The logic of continua is the most important branch of the logic of relatives, and mathematics, especially geometrical topic, or topical geometry, has its development retarded from the lack of a developed logic of continua.

Literature: relatives have, since Aristotle, been a recognized topic of logic. The first germ of the modern doctrine appears in a somewhat trivial remark of ROBERT LESLIE ELLIS. DE MORGAN did the first systematic work in his fourth memoir on the syllogism in 1860 (Cambridge Philos. Trans., x. 231–358); he here sketched out the theory of dyadic relations. C. S. PEIRCE, in 1870, extended Boole's algebra so as to apply to them, and after many attempts produced a good general algebra of logic, together with another algebra specially adapted to dyadic relations (Studies in Logic, by members of the Johns Hopkins University, 1883, Note B, 187–203). SCHRÖDER developed the last in a systematic manner (which brought out its glaring defect of involving hundreds of merely formal theorems without any significance, and some of them quite difficult) in the third volume of his Exakte Logik (1895). SCHRÖDER'S work contains much else of great value. PEIRCE has published only three papers since 1883, one of which appeared in the Amer. J. of Math., vii. (1885) 180–202, and the other two in the Monist, vii. (1896–7) 19–40, 161–217. An important work in which relations are treated graphically is A. B. KEMPE'S Theory of Mathematical Form, published in the Philos. Trans. for 1890. Other workers are JOSEPH JOHN MURPHY, ALEXANDER MACFARLANE, GIUSEPPE PEANO. GEORG CANTOR, RICHARD DEDEKIND, and others have treated relations of quantity, and their writings—especially DEDEKIND'S book, Essays on the Theory of Number (Eng. trans., 1901)—are particularly recommended to students of philosophy. Translations of parts of some of CANTOR'S memoirs into most puzzling French are given in the Acta Mathematica, ii; the Math. Annalen (xlvi and xlix) contain others of great importance; and CANTOR especially addresses students of philosophy in his Zur Lehre vom Transfiniten Erste Abth., (1890). This brochure consists of papers originally printed in the Zeitsch. f. Philos. u. philos. Krit. See also V. B. RUSSELL, Sur la logique des relations, in Revue de Mathématiques, vii (1901); WHITEHEAD, in recent numbers of the Amer. J. of Math. (O.S.P.)

Relativity: Ger. *Relativität*; Fr. *relativité*; Ital. *relatività*. That element in the determination of a thing or object which arises from its RELATION (q.v.) to other things or objects. See the following topics. (J.M.B.)

Relativity (affective). The liability of affective states to modification by other affective states.

Applied (1) to pleasure and pain, which are said to be relative to each other; an extreme form, holding that pleasure is only absence of pain (cf. the literature of PAIN AND PLEASURE, the 'relativity theory'); (2) to emotions, considered as liable to modification from one another. See CONTRAST (affective). (J.M.B.)

Relativity (in psychology, law of): Ger. *Beziehungsgesetz*, *Gesetz der Relativität*; Fr. *loi de relativité*; Ital. *legge di relazione* (or *relatività*). (1) The law that every phase of experience is influenced by every other phase of the experience of the moment, and also by the whole past history of consciousness.

It is employed by Wundt to explain Weber's law, certain geometrical optical illusions, visual contrast, temperature adaptation, &c. Experimental psychology has, however, in general preferred to look to physiology for the conditions of such mental facts or laws (Wundt, *Physiol. Psychol.*, 4th ed., i. 393, 397, 416, 591; *Human and Animal Psychol.*, 62, 119, 264). In Wundt's *Grundriss* (1896) the law of relativity assumes a threefold form: the law of psychical resultants, the law of relations, and the law of psychical contrasts. This theory is known as that of the 'relativity of sense qualities.' Cf. CONTRAST (various topics). (E.B.T.)

(2) The theory defined under RELATIVITY OF KNOWLEDGE (2).

Relativity of Knowledge: Ger. (1) *Relativismus*, (2) *Relativität der Erkenntniss*; Fr. *relativité de la connaissance*; Ital. *relatività della conoscenza*. (1) This term seems most properly to denote the theory that all human knowledge is relative to the human mind, in the sense that we can only know, of things, the effects which they produce upon our minds, and not what they themselves are like. Cf. EPISTEMOLOGY.

The theory is thus, as it stands, doubly self-contradictory, since it combines the proposition (*a*) that each of us can know nothing but what is in his own mind, whence it follows that he cannot know that anything but his own mind exists; with the propositions (*b*) that we do know that what is in our minds is an effect of other things; and (*c*) that this is true of *us*, i. e. that more than one

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