Syllogistic (argumentation; also used as a noun). SYLLOGISM (q.v.).

Symbol [Gr. σύμβολον, a conventional sign, from σύν + βάλλειν, to throw]: Ger. *Symbol*; Fr. *symbole*; Ital. *simbolo*. (1) A SIGN (q.v.) which is constituted a sign merely or mainly by the fact that it is used and understood as such, whether the habit is natural or conventional, and without regard to the motives which originally governed its selection.

Σύμβολον is used in this sense by Aristotle several times in the *Peri hermeneias*, in the *Sophistici Elenchi*, and elsewhere.

(2) An algebraic character. (C.S.P.)

Symbol (and **Symbolic**) [Gr. σύν + βάλλειν, to put together, compare]: Ger. (*symbolisch*); Fr. (*symbolique*); Ital. (*simbolica*). (1) An object which stands for some other object or idea; the former is said to be 'symbolic' of the latter. Cf. SIGN, and SIGN-MAKING FUNCTION.

(2) In aesthetics, an object which, apart from its own immediate and proper significance, suggests also another, especially a more ideal content which it cannot perfectly embody.

The symbol may be either natural: as light is a symbol of truth; or traditional and conventional: as the cross is a symbol of sacrifice.

The conception of art as symbolic goes back at least to Plotinus, but the term seems to have come into general aesthetic currency through Goethe and Schlegel—the latter declaring it to be (in sense 1, above) the essence of all art. Hegel made the symbolic in sense (2) the principle of oriental as compared with Greek art. Vischer laid special stress on the symbolic (significant) character of art, as against the Formalists. Recently, the psychology of symbolization has received special treatment. Fechner explained it as association. Others have considered it as an investiture of the object with the observer's own idea and feeling in a more intimate manner than is implied by the term association, and have sought for terms expressing this, as 'mitfühlen,' feeling with (Lotze), 'einfühlen,' feeling into (R. Vischer, Fr. Vischer), a lending or animating (Leihen, Beseelung; Fr. Vischer), fusion (Verschmelzung; Volkelt). According to Lotze we live over again in the object the motion to produce it, &c. Groos (*Play of Man*, Eng. trans., 31) makes eye-movements and other 'inner imitations' 'symbolic' of the real movements of imitation. See SYMPATHY (aesthetic).

Literature (to 2): HEGEL, Aesthetik, ii. Th., i. Abth.; STERN, Einfühlung u. Association in d. neu. Aesth. (1898); FECHNER, Vorschule d. Aesth., ii; LOTZE, Gesch. d. Aesth., 74 ff.; FR. VISCHER, Aesthetik; Krit. Gänge, v, vi; and Das Symbol, Altes u. Neues (1889); R. VISCHER, Über d. optische Formgefühl (1873); VOLKELT, Der Symbolbegriff in d. neuesten Aesth. (1876); LIPPS, Raumästhetik u. geometrisch-optische Täuschungen (1897); VOLKELT, Zeitsch. f. Philos., cxiii. 161–79; STERN, ibid., cxv. 193–203; KÜLPE, Zeitsch. f. wiss. Philos., xxiii. 145–83; TURNARKIN, Arch. f. Gesch. d. Philos., xii. 257–89; FERRERO, I simboli (1892). Cf. also FORM, BALANCE, SYMMETRY. (J.H.T.)

Symbolic Function: no foreign equivalents in use. The function whereby a mental result primarily referring to one set of objects is transferred to another set of objects; the first set is said to be symbolic of the second.

SYMBOL (q.v.) is frequently used in a very wide sense as equivalent to any kind of sign. But it seems desirable to limit its application in psychology to cases in which the sign is provisionally substituted for the thing symbolized. Words are not substitute signs in this sense; they are means by which we attend to what is signified, not themselves objects of attention. Cf. SIGN-MAKING FUNCTION, and SIGN (for a more special meaning of symbol). (G.F.S.)

Symbolic Logic or **Algebra of Logic**: Ger. *Algebra der Logik*; Fr. *logique symbolique ou algorithmique, algèbre de la logique*; Ital. *logica simbolica*. Symbolic logic is that form of logic in which the combinations and relations of terms and of propositions are represented by symbols, in such a way that the rules of a calculus may be substituted for actively conscious reasoning.

An algebra of logic enables us to disengage from any subject-matter the formal element which gives its necessary (apodictic) force to reasoning; it is therefore nothing but an exact logic, that is to say, the complete realization of the purpose of formal logic (cf. PROPOSITION). The ordinary formal logic has, from the earliest times, substituted symbols (viz. the letters of the alphabet) for significant terms, and has thus added much to the facility with which the validity of arguments can be tested; symbolic logic goes a step further, and adds symbols to stand for combinations of terms, or functions of terms, and statements of relations between terms. The aid which is thus given to logic, not only in the carrying out of complicated trains of

reasoning, but also in the exact analysis of the various steps involved, is very great.

Several systems of symbolic logic have been proposed within the last half-century (see literature). We shall here describe only one—that of Boole, as reformed and developed by Schröder, Peirce, and others. This system is not based exclusively upon the consideration of the extension (application) of terms and of propositions, but covers all relations of intension (SIGNIFICATION, q.v.) as well. It is, however, more convenient, when formulae are to be expressed in words, to use the language of one or the other of these two parallel interpretations exclusively; that of the application-interpretation will be used in what follows.

Throughout symbolic logic there is an exact analogy between terms and propositions, so that the same theorems (or formulae) apply to both; it is not a case of two parallel systems (a calculus of concepts and a calculus of propositions), but of a single system susceptible of a double interpretation. In what follows, the letters of the alphabet stand for either concepts or propositions[1].

The algebra of logic rests upon two *relations*—that of inclusion (or subsumption, or sufficient condition) and that of equality, of which the first only is fundamental—and upon three operations—aggregation (or logical addition), composition (or logical multiplication, as it has been unfortunately named, upon a false analogy), and negation. Of the three operations, negation together with either of the other two would suffice for the algebra (though facility of expression is greatly increased by admitting all three of them); hence one relation (or form of statement) and one operation, together with negation (applied not only to terms but also to the assumed form of statement and to the assumed operation), are all that are absolutely essential to the building up of the theory.

The relation of inclusion, which is written $a \leqslant b$, signifies that the class a constitutes a part (or it may be the whole) of the class b, or that the quality-complex a is indicative of the quality-complex b, or that the statement a involves the statement b. Conceptual Interpretation: The a's are all b's; Propositional Interpretation: If a is true b is true, or, a entails b. The relation of equality, or identity, which is written $a = b$, signifies, for one thing, that the two classes a and b are identical (made up out of the same elements). It may be defined as equivalent to the system of two inverse inclusions

$$(a \leqslant b)(b \leqslant a);$$

C.I.: All a is b and all b is a; P.I.: a entails b and b entails a. In the case of propositions, logical equality is called *equivalence*. Multiplication and addition are thus defined in terms of classes: the sum of two classes is the class which contains all the elements of each (without repetition); the product is the class which contains all the elements which are common to both. Formally these operations may be defined as follows:

$$(a \leqslant c)(b \leqslant c) = (a + b \leqslant c);$$

C.I.: If a is c and b is c, what is either a or b is c, and conversely; P.I.: If a implies c and b implies c, whatever implies either a or b implies c, and conversely.

$$(c \leqslant a)(c \leqslant b) = (c \leqslant ab);$$

C.I.: If c is a and c is b, c is a and b, and conversely; P.I.: If c implies a and c implies b, c implies both a and b, and conversely. It will be seen that the signs $+$ and $\times$ (understood in the form ab) correspond to a certain extent to the conjunctions *or* and *and*, but not completely; for instance, $a + b \leqslant c$ must be read '*a and b* are *c*,' but by throwing the first member of this inclusion into a subordinate predicate (which can always be done without change of meaning) it may be read 'What is *a or b* is *c*.' The inclusion $ab \leqslant c$ can be read '*a* which is *b* is *c*,' or '*b* which is *a* is *c*,' or 'What is *a* and *b* is *c*.'

It is necessary to define at once two special terms which play an important rôle in symbolic logic, the logical *zero* (0) and the logical *everything* (∞ or 1). They are defined formally as follows:

$$0 \leqslant x, \quad x \leqslant 1,$$

where x stands for any term whatever, or for any proposition whatever. In the conceptual interpretation, 1 is *everything* which *exists*, or the universe of discourse, and 0 is *nothing*, or *the non-existent*; in the propositional interpretation, 1 is the aggregate of those states of things which *occur*, or are *true*, and 0 is the *false*, or the *non-occurrent*. The special terms may equally well be defined as follows:

$$x + 0 \leqslant x, \quad x \leqslant x \times 1;$$

we should then say that 0 is that term which, when added to any term, makes it no greater than it was before, and that 1 is that term which, when compounded with any term, makes it no less than it was before. From either of these pairs of definitions the other

[1] Abbreviations: C. I. = conceptual interpretation; I. = propositional interpretation.

pair follows at once; the following formulae are also evident:

$$0 \leqslant 0, \quad 0 \leqslant 1, \quad 1 \leqslant 1,$$
$$(x \leqslant 0) = (x = 0), \quad (1 \leqslant x) = (1 = x),$$
$$x + 0 = x, \quad x \times 1 = x.$$

The third operation of exact logic is negation. It is indicated by a horizontal line placed above the term or the expression to be denied; $\bar{a}$ signifies non-a; $\bar{a} \overline{\leqslant} \bar{b}$ is the denial of All a is b. (In this last case the sign of negation may be equally well placed upon the copula; $a \overline{\leqslant} b$ means Not all a is b.) Negation may be defined formally by the two following statements: $a\bar{a} \leqslant 0$, $1 \leqslant a + \bar{a}$, which translate respectively the principles of 'contradiction' (or mutual exclusion) and of 'excluded middle' (or conjoint exhaustion—see LAWS OF THOUGHT). C.I.: a which is non-a is non-existent, Everything is a or non-a; P.I.: The statements a and non-a cannot both be true at once, What is possible is that a is true or that non-a is true (i.e. that a is false). It can be proved that the negative as thus defined is unequivocal, i.e. that the term non-a is unique.

The propositions of logic may all be deduced from the definitions and a limited number of principles, or axioms, which are independent and irreducible; among them are the principle of identity: $a \leqslant a$ (C.I.: All a is a; P.I.: If a is true a is true), which has for a corollary $a = a$; and the principle of the syllogism:

$$(a \leqslant b)(b \leqslant c) \leqslant (a \leqslant c)$$

(C.I.: If a is b and if b is c, then a is c; P.I.: If a implies b and b implies c, then a implies c). The operations of multiplication and addition are subject to the commutative law,

$$a + b = b + a, \quad ab = ba,$$

the associative law,

$$(a + b) + c = a + (b + c), \quad (ab)c = a(bc),$$

and to the special law of tautology,

$$a + a = a, \quad aa = a.$$

The law of absorption,

$$a + ab = a, \quad a(a + b) = a,$$

can be proved, but the law of distribution,

$$a(b + c) = ab + ac, \quad a + bc = (a + b)(a + c),$$

it is not possible to demonstrate without the assumption of an additional principle, or axiom, namely

$$a(b + c) \leqslant ab + ac.$$

The distributive law has for corollaries the following formulae:

$$ab + cd = (a + c)(b + c)(a + d)(b + d),$$
$$(a + b)(c + d) = ac + bc + ad + bd.$$

These formulae, as well as all those already given, show that there is a perfect correlation, or duality, between addition and multiplication, which consists in the fact that the signs $+$ and $\times$ may be interchanged upon the condition of interchanging at the same time the special terms 0 and 1, and inverting the sign of inclusion, $\leqslant$.

The following formulae may also be demonstrated:

$$(a \leqslant b)(c \leqslant d) \leqslant (ac \leqslant bd)$$
$$\leqslant (a + c \leqslant b + d)$$
$$(a = b)(c = d) \leqslant (ac = bd)$$
$$\leqslant (a + c = b + d);$$

these enable us to combine (but not without loss) several inclusions or equalities by either adding or multiplying them member by member (as in algebra). It is also possible to add a common term to each member of an inclusion or an equation (but not to take one away) and to introduce a common term as a factor (but not to remove one).

The operation of negation adds important properties to the algebra, of which the principals are: *the law of double negation*, $\bar{\bar{a}} = a$ (C.I.: Non-non-a is identical with a; P.I.: To deny the denial of a statement is the same as to affirm it); the *formulae of De Morgan*,

$$\overline{ab} = \bar{a} + \bar{b}, \quad \overline{a + b} = \bar{a}\bar{b},$$

which enable us to distribute the process of denying upon the elements of a sum or of a product (and which illustrates the duality mentioned above), and the principle of contraposition,

$$(a \leqslant b) = (\bar{b} \leqslant \bar{a}),$$
$$(a \overline{\leqslant} b) = \bar{b} \overline{\leqslant} \bar{a})$$

(C.I.: 'All a is b' is the same thing as 'All non-b is non-a'; 'Not all a is b' is the same thing as 'Not all non-b is non-a'; P.I.: 'If a is true b is true' has the same validity as 'If b is false a is false'; that 'The truth of a does not entail the truth of b' is equivalent to saying that 'The falsity of b does not entail the falsity of a'). As a corollary to this we may add

$$(a = b) = (\bar{a} = \bar{b}).$$

The principle of contraposition is merely a special case of the principle of TRANSPOSITION (q.v.), that is, of

$$(ac \leqslant b + d) = (a\bar{d} \leqslant b + \bar{c}),$$
$$(ac \overline{\leqslant} b + d) = (a\bar{d} \overline{\leqslant} b + \bar{c}),$$

which may be stated thus: an element of a sum in a predicate is the same thing as its negative as a factor of the subject, both in the universal and in the particular statement in terms of this copula. (But the opposite relation does not hold—an element of a sum cannot be introduced in this way into a subject nor a factor into a predicate.)

The formulae for the addition and the multiplication of 0 and 1,

$$0 + x = x, \quad 1 + x = 1,$$
$$0 \times x = 0, \quad 1 \times x = x,$$

lead to the *formulae of development*, which were given by Boole,

$$x = x(a + \bar{a})(b + \bar{b}) \ldots$$
$$= xab \ldots + xa\bar{b} \ldots + x\bar{a}b \ldots + x\,\bar{a}\bar{b} \ldots,$$
$$x = x + a\bar{a} + b\bar{b} + \ldots$$
$$= (x + a + b)(x + a + \bar{b})(x + \bar{a} + b) \ldots,$$
$$0 = (a + b)(a + \bar{b})(\bar{a} + b)(\bar{a} + \bar{b}),$$
$$1 = ab + a\bar{b} + \bar{a}b + \bar{a}\bar{b}$$
$$= abc + ab\bar{c} + a\bar{b}c + \ldots,$$

and so for any number of simple terms, a, b, c, ... (The terms of the development of 1 are called its *constituents*.)

To Boole is due also the formula for the development of a function in terms of any variable, or unknown quantity, x, which it contains: $F(x) = F(1)x + F(0)\bar{x}$, $F(1)$ being what $F(x)$ becomes for $x = 1$, and $F(0)$ being what $F(x)$ becomes for $x = 0$. Hence one of the normal forms for a logical statement (in one unknown quantity) is

$$ax + b\bar{x} \leqslant 0,$$

which may equally well be written (since $0 \leqslant F(x)$ is always true, no matter what $F(x)$ may be),

$$ax + b\bar{x} = 0,$$

or,

$$\bar{a}x + \bar{b}\bar{x} = 1.$$

In order to reduce the problems of logic to inclusions or equations of this form, it is necessary to apply to the premises (into which the verbal data have been translated) the preceding formulae of transformation, and to bring them thus into forms in which the second member is either 0 or 1; they are then to be combined in accordance with the following formulae:

$$(a = 0)(b = 0) = (a + b = 0),$$
$$(a = 1)(b = 1) = (ab = 1),$$

until there is only a single equation to be resolved, of one or the other of the two forms,

$$ax + b\bar{x} = 0, \quad (a + x)(b + \bar{x}) = 1.$$

We shall confine ourselves to the treatment of the first of these two forms (the reader can easily translate it, step by step, into the treatment for the second form). The equation is equivalent to this system of two inclusions,

$$ax \leqslant 0, \quad b\bar{x} \leqslant 0,$$

or,

$$x \leqslant \bar{a}, \quad b \leqslant x,$$

that is to say, to $b \leqslant x \leqslant \bar{a}$, (S_1)

whence $b \leqslant \bar{a}$, or, $ab \leqslant 0$. (S_2)

Thus the solution is, in words, x contains b and is contained in $\bar{a}$, or, as it can be otherwise expressed,

$$x = \bar{a}x + b\bar{x} \text{ (Poretsky)},$$
$$x = \bar{a}u + b\bar{u} \text{ (Schröder)}.$$

(In the last expression u is a purely arbitrary term.) The two extreme values of x for $u = 1$ and $u = 0$ are $x = b$, $x = \bar{a}$. But the solution of the equation in terms of x is given completely in (S_1), and (S_2) contains all that is involved in the premises independently of x, that is, it is the resultant which remains after the elimination of x. It is also the condition for the resolvability of the original expression.

[But the problem of eliminating x appears in a still more interesting form if we equate to 0 a *sum* and to 1 a *product* of functions of x and $\bar{x}$, if we write, that is, for the canonical form of the equations to be resolved,

$$(a + x)(b + \bar{x}) = 0, \quad ax + b\bar{x} = 1,$$

instead of those just given. The rule for the elimination of the quantity to be discarded is then exactly the same for both of these expressions; it is simply: *erase it*. (Of course, if either a or b is zero in the left-hand form or 1 in the right-hand form, x cannot be eliminated, for we have then only one premise instead of two.) Moreover, this same rule applies to the elimination of the unknown quantity in the particular propositions,

$$ax + b\bar{x} \neq 0, \quad 1 \neq (a + x)(b + \bar{x});$$

they give, respectively, $a + b \neq 0$, $1 \neq ab$. The argument is here (1) If some a is x or else some b is non-x, then in any case Something is either a or b; and (2) If not everything is at once either a or x and also b or $\bar{x}$, then, all the more, Not everything is at once a and b. The first of these two forms is probably more convincing intuitively than the second.—C.L.F.]

The common syllogism, when universal, is a particular case of the equations just discussed, if a and b are simple terms, instead of expressions of any degree of complexity. These formulae do not, of course, constitute a demonstration of the principle of the syllogism, for they depend upon it.

The formulae of symbolic logic have usually been developed in terms of the non-symmetrical affirmative copula, $a \leqslant b$, and its denial, $a \overline{\leqslant} b$; this method is the best in point of naturalness, but either of the universal symmetrical copulas (see PROPOSITION, in loc.) combined with either of the corresponding particular copulas gives an algebra which has great advantage in point of conciseness; a single formula takes the place, throughout, of the dual pair of formulae of Schröder (see *Studies in Logic*, by members of the Johns Hopkins University.)

Exact logic does not admit the deduction from the universal affirmative proposition, $a \leqslant b$, of 'Some b is a' nor of 'Some a is b'; for the proposition $a \leqslant b$ does not imply the existence of a, since it is true (no matter what may be the meaning of b) for the value $a = 0$, while 'Some a is b' and 'Some b is a' $(ab \,\overline{\leqslant}\, 0)$ do imply the existence of a, since

$$(a \leqslant 0) + (b \leqslant 0) \leqslant (ab \leqslant 0).$$

But these two deductions are permissible whenever we are in possession of the additional information that a exists, or that $a \,\overline{\leqslant}\, 0$.

For we have $(a \leqslant b) = (a\bar{b} \leqslant 0)$.

Now $(ab \leqslant 0)(a\bar{b} \leqslant 0) \leqslant (a \leqslant 0)$,
Whence, by the *principle of transposition*, we have $(a\bar{b} \leqslant 0)(a \,\overline{\leqslant}\, 0) \leqslant (ab \,\overline{\leqslant}\, 0)$.

It is by means of this principle of transposition that Mrs. Ladd-Franklin has reduced the traditional fifteen valid moods of the syllogism, or the 8,192 $(= 16 \times 16 \times 16 \times 2)$ valid syllogisms which are possible if the full scheme of propositions—as Everything is a or b, Not all but a is b, &c.—is taken account of, to the single formula

$$(ab = 0)(\bar{b}c = 0)(ac \neq 0) \leqslant 0,$$

which may be called an Antilogism, and to which corresponds either the universal or the particular syllogism (in all its forms) according as one or another of these three incompatible propositions is transferred to the conclusion,

$$(ab = 0)(\bar{b}c = 0) \leqslant (ac = 0),$$
$$(ab = 0)(ac \neq 0) \leqslant (\bar{b}c \neq 0).$$

(See Schröder, *Algebra d. Logik*, § 43, and E. Müller, *Ueber d. Algebra d. Logik*, ii. 19.) Cf. PROPOSITION.

The theorems given hitherto hold equally for concepts and for propositions. But there is a special set of theorems for such propositions as are either *always true* or *always false*. These theorems follow from the two following formulae, which constitute the definition of propositions of this kind,

$$A = (1 \leqslant A), \quad A_1 = (A \leqslant 0)$$

(with capital letters it is convenient to write a dash for the sign of negation), or, as they may also be written,

$$A = (1 = A), \quad A_1 = (A = 0).$$

These propositions, that is to say, have only two values, 0 and 1. Propositions of variable value are such as contain one or several indeterminate quantities $(x, y, z, \ldots)$, for some values of which the propositions are true, for others false. They have an intermediate extension (Gültigkeitsbereich) between 0 and 1, measured mathematically by their probability.

The following formulae hold for propositions of constant value:

$$(1 = A + B) = (1 = A) + (1 = B),$$
$$(AB = 0) = (A = 0) + (B = 0),$$
$$(A \leqslant B) = (A_1 + B).$$

In the last of these equations we permit ourselves to write (following a peculiarity of language) simply $A_1 + B$ instead of $1 \leqslant A_1 + B$. To take an example, the proposition 'u is v implies that x is y' becomes, upon transposition of the first member, 'What is possible is that u is not v or else that x is y,' a statement which we are in the habit of using in the apocopated form, 'u is not v or x is y.' This abbreviation amounts, in the algebra, to the convention that whatever expression, a, shall be simply written upon our sheet of paper shall be understood to have the force of the statement $1 \leqslant a$; we might equally well, if we had adopted the negative copula, agree that whatever, x, is written upon the paper has the force of $x \leqslant 0$. Neither procedure would be permissible if particular propositions, the denials of universal propositions, were to be treated at the same time, but by hypothesis we are here dealing only with statements which have no other values than 0 and 1, that is, which are universal.

We have, again,

$$(A = B) = AB + A_1B_1,$$
$$(A = B_1) = AB_1 + A_1B,$$

and also, $(AB \leqslant C) = (A \leqslant C) + (B \leqslant C)$,

$$(C \leqslant A + B) = (C \leqslant A) + (C \leqslant B),$$

and the theorem due to Mr. Peirce,

$$(AB \leqslant C) = [A \leqslant (B \leqslant C)]$$
$$= [B \leqslant (A \leqslant C)],$$

and finally the principle of hypothetical reasoning, direct and inverse,

$$(A \leqslant B)A \leqslant B, \quad (A \leqslant B)B_1 \leqslant A_1.$$

For propositions of variable value another notation may be used. Let $f(x)$ be a logical function containing the variable x, which is capable of taking the several values $a, b, c, \ldots$; and let

$$\Sigma_x f(x) = f(a) + f(b) + f(c) + \ldots,$$
$$\Pi_x f(x) = f(a) \times f(b) \times f(c) \times \ldots;$$

then the equations

$$\Sigma_x f(x) = 0, \quad \Pi_x f(x) = 0,$$

signify, the first, that for every one of the values of x the equation $f(x) = 0$ is satisfied; the second, that for some one at least of its values the equation is satisfied. The formula for the solution of equations given above becomes, in this notation,

$$(ax + b\bar{x} = 0) = (ab = 0)\,\Sigma_u(x = \bar{a}u + b\bar{u}),$$

which means that if the equation $ax + b\bar{x} = 0$

holds, then on the one hand $ab = 0$, and on the other hand for every one of the values of u, $x = \bar{a}u + b\bar{u}$ satisfies the equation, and reciprocally.

Symbolic logic, it will be seen, constitutes a real algebra, which has its own laws; it gives rise to a theory of equations and of inequalities which has not yet been fully worked out. It also serves as an introduction to a more general logic—the logic of RELATIVES (q. v.)—of which it is a particular case. This latter was foreseen by Leibniz, prepared for by De Morgan, founded by Peirce, and developed by Schröder.

Besides the system of symbolic logic here developed, to which the writings of Johnson, Whitehead, Poretsky, Mitchell, and Mrs. Ladd-Franklin have contributed, the principal systems that have been proposed are (1) that of Jevons, which consists in forming all possible combinations of positive and negative factors (the constituents of Boole), suppressing those which are annulled by the given premises, and reuniting the remaining combinations to form the solution of the problem (an operation which may be facilitated by diagrams and by a logical machine); (2) that of Peirce, whose method consists in separating the combined data up into the product (instead of the sum) of a function of x and of non-x, and in eliminating x by means of the formula

$$(ax \leqslant b)(c\bar{x} \leqslant d) \leqslant (ac \leqslant b + d);$$

(3) that of MacColl, which consists in considering propositions alone as the elements of reasoning, and in assigning to them three distinct values: ϵ (1), η (0), θ (neither 1 nor 0)—this method is particularly adapted to questions of probability and to certain questions of mathematics (the calculus of the limits of a multiple integral), which in fact gave rise to it; (4) that of Peano, the object of which is to analyse and to verify the propositions of mathematics, and which employs, besides the logical symbols (necessarily different from the preceding, but nearly equivalent to them), symbols for mathematical notions and relations. (L.C.–C.L.F.)

If symbolic logic be defined as logic—for the present only deductive logic—treated by means of a special system of symbols, either devised for the purpose or extended to logical from other uses, it will be convenient not to confine the symbols used to algebraic symbols, but to include some graphical symbols as well.

[The reader will observe that the symbols adopted for the dictionary are in some measure departed from in what follows.—J.M.B.]

The first requisite to understanding this matter is to recognize the purpose of a system of logical symbols. That purpose and end is simply and solely the investigation of the theory of logic, and not at all the construction of a calculus to aid the drawing of inferences. These two purposes are incompatible, for the reason that the system devised for the investigation of logic should be as analytical as possible, breaking up inferences into the greatest possible number of steps, and exhibiting them under the most general categories possible; while a calculus would aim, on the contrary, to reduce the number of processes as much as possible, and to specialize the symbols so as to adapt them to special kinds of inference. It should be recognized as a defect of a system intended for logical study that it has two ways of expressing the same fact, or any superfluity of symbols, although it would not be a serious fault for a calculus to have two ways of expressing a fact.

There must be operations of transformation. In that way alone can the symbol be shown determining its interpretant. In order that these operations should be as analytically represented as possible, each elementary operation should be either an insertion or an omission. Operations of commutation, like $xy \therefore yx$, may be dispensed with by not recognizing any order of arrangement as significant. Associative transformations, like $(xy)z \therefore x(yz)$, which is a species of commutation, will be dispensed with in the same way; that is, by recognizing an equiparant as what it is, a symbol of an unordered set.

It will be necessary to recognize two different operations, because of the difference between the relation of a symbol to its object and to its interpretant. Illative transformation (the only transformation relating solely to truth that a system of symbols can undergo) is the passage from a symbol to an interpretant, generally a partial interpretant. But it is necessary that the interpretant shall be recognized without the actual transformation. Otherwise the symbol is imperfect. There must, therefore, be a sign to signify that an illative transformation would be possible. That is to say, we must not only be able to express 'A therefore B,' but 'If A then B.' The symbol must, besides, separately indicate its object. This object must be indicated by a sign, and the relation of this to the significant element of the symbol is that both are signs of the same object. This is an equiparant, or commutative relation. It is therefore necessary to have an operation combining two symbols as referring to the same object. This, like the other operation, must have its actual and its potential state. The former makes the symbol a proposition 'A is B,' that is, 'Something A stands for, B stands for.' The latter expresses that such a proposition *might* be expressed, 'This stands for something which A stands for and B stands for.' These relations might be expressed in roundabout ways; but two operations would always be necessary. In Jevons's modification of Boole's algebra the two operations are aggregation and composition. Then, using non-relative terms, 'nothing' is defined as that term which aggregated with any term gives that term, while 'what is' is that term which compounded with any term gives that term. But here we are already using a third operation; that is, we are using the relation of equivalence; and this is a composite relation. And when we draw an inference, which we cannot avoid, since it is the end and aim of logic, we use still another. It is true that if our purpose were to make a calculus, the two operations, aggregation and composition, would go admirably together. Symmetry in a calculus is a great point, and always involves superfluity; as in homogeneous co-ordinates and in quaternions. Superfluities which bring symmetry are immense economies in a calculus. But for purposes of analysis they are great evils.

A proposition *de inesse* relates to a single state of the universe, like the present instant. Such a proposition is altogether true or altogether false. But it is a question whether it is not better to suppose a general universe, and to allow an ordinary proposition to mean that it is sometimes or possibly true. Writing down a proposition under certain circumstances asserts it. Let these circumstances be represented in our system of symbols by writing the proposition on a certain sheet. If, then, we write two propositions on this same sheet, we can hardly resist understanding that both are asserted. This, then, will be the mode of representing that there is something which the one and the other represent—not necessarily the same quasi-instantaneous state of the universe, but the same universe. If writing A asserts that A may be true, and writing B that B may be true, then writing both together will assert that A may be true and that B may be true.

By a *rule* of a system of symbols is meant a permission under certain circumstances to make a certain transformation; and we are to recognize no transformations as elementary except writing down and erasing. From the conventions just adopted, it follows, as Rule I, that *anything written down may be erased, provided the erasure does not visibly affect what else there may be which is written along with it.*

Let us suppose that two facts are so related that asserting the one gives us the right to assert the other, because if the former is true, the latter must be true. If A having been written, we can add B, we may then, by our first rule, erase A; and consequently A may be transformed into B by two steps. We shall need to express the fact that writing A gives us a right, under all circumstances, to add B. Since this is not a reciprocal relation, A and B must be written differently; and since neither is positively asserted, neither must be written so that the other could be erased without affecting it. We need some place on our sheet upon which we can write a proposition without asserting it. The present writer's habit is to cut it off from the main sheet by enclosing it within an oval line; but in order to facilitate the printing, we will here enclose it in square brackets. In order, then, to express that 'If A can under any circumstances whatever be true, B can under some circumstances be true,' we must certainly enclose A in square brackets. But what are we to do with B? We are not to assert positively that B can be true; yet it is to be more than hypothetically set forth, as A is. It must certainly, in some fashion, be enclosed within the brackets; for were it detached from the brackets, the brackets with their enclosed A could, by Rule I, be erased; while in fact the dependence upon A cannot be omitted without danger of falsity. It is to be remarked that, in case we can assert that 'If A can be true, B can be true,' then, *a fortiori*, we can assert that 'If both A and C can be true, B can be true,' no matter what proposition C may be. Consequently, we have, as Rule II, that, *within brackets already written, anything whatever can be inserted.* But the fact that 'If A can be true, B can be true' does *not* generally justify the assertion 'If A can be true, both B and D are true'; yet our second rule would imply that, unless the B were cut off, in some way, from the main field within the brackets. We will therefore enclose B in parentheses, and

express the fact that 'If A can be true, B can be true' by

$$[A\,(B)] \text{ or } [(B)\,A] \text{ or } \begin{bmatrix} A \\ (B) \end{bmatrix}, \text{ \&c.}$$

The arrangement is without significance. The fact that 'If A can be true, both B and D can be true,' or $[A\,(BD)]$, justifies the assertion that 'If A is true B is true,' or $[A\,(B)]$. Hence the permission of Rule I may be enlarged, and we may assert that anything unenclosed or enclosed both in brackets and parentheses can be erased if it is separate from everything else. Let us now ask what $[A]$ means. Rule II gives it a meaning; for by this rule $[A]$ implies $[A\,(X)]$, whatever proposition X may be. That is to say, that $[A]$ can be true implies that 'If A can under any circumstances be true, then anything you like, X, may be true.' But we may like to make X express an absurdity. This, then, is a *reductio ad absurdum* of A; so that $[A]$ implies, for one thing, that A cannot under any circumstances be true. The question is, Does it express anything further? According to this, $[A\,(B)]$ expresses that $A\,(B)$ is impossible. But what is this? It is that A can be true while something expressed by (B) can be true. Now, what can it be that renders the fact that 'If A can ever be true, B can sometimes be true' incompatible with A's being able to be true? Evidently the falsity of B under all circumstances. Thus, just as $[A]$ implies that A can never be true, so (B) implies that B can never be true. But further, to say that $[A\,(B)]$, or 'If A is ever true, B is sometimes true,' is to say no more than that it is impossible that A is ever true, B being never true. Hence, the square brackets and the parentheses precisely deny what they enclose. A logical principle can be deduced from this: namely, if $[A]$ is true $[A\,(X)]$ is true. That is, if A is never true, then we have a right to assert that 'If A is ever true, X is sometimes true,' no matter what proposition X may be. Square brackets and parentheses, then, have the same meaning. Braces may be used for the same purpose. Moreover, since two negatives make an affirmative, we have, as Rule III, that *anything can have double enclosures added or taken away, provided there be nothing within one enclosure but outside the other.* Thus, if B can be true, so that B is written, Rule III permits us to write $[(B)]$, and then Rule II permits us to write $[X\,(B)]$. That is, if B is sometimes true, then 'If X is ever true, B is sometimes true.' Let us make the apodosis of a conditional proposition itself a conditional proposition. That is, in $(C\,\{D\})$ let us put for D the proposition $[A\,(B)]$. We thus have $(C\,\{[A\,(B)]\})$. But, by Rule III, this is the same as $(C\,A\,(B))$.

All our transformations are analysed into insertions and omissions. That is, if from A follows B, we can transform A into AB and then omit the B. Now, by Rule I, from AB follows A. Treating this in the same way, we first insert the conclusion and say that from AB follows ABA. We thus get as Rule IV that *any detached portion of a proposition can be iterated.*

It is now time to reform Rule II so as to state in general terms the effect of enclosures upon permissions to transform. It is plain that if we have written $[A\,(B)]\,C$, we can write $[A\,(BC)]\,C$, although the latter gives us no right to the former. In place, then, of Rule II we have:

Rule II (amended). *Whatever transformation can be performed on a whole proposition can be performed upon any detached part of it under additional enclosures even in number, and the reverse transformation can be performed under additional enclosures odd in number.*

But this rule does not permit every transformation which can be performed on a detached part of a proposition to be performed upon the same expression otherwise situated.

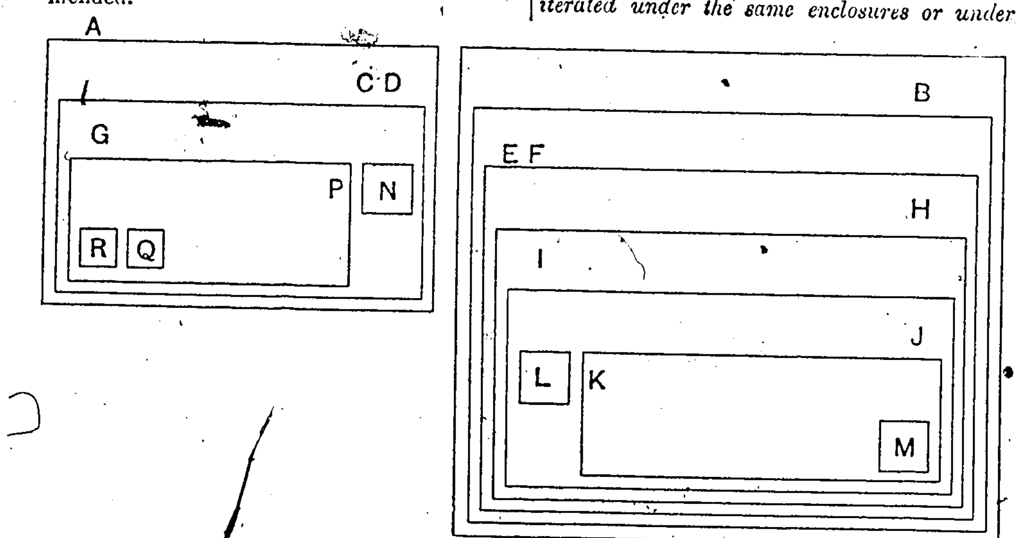
Rule IV permits, by virtue of Rule II (amended), all iteration under additional enclosures and erasure of a term inside enclosures if it is iterated outside some of them.

We can now exhibit the *modi tollens et ponens*. Suppose, for example, we have these premises: 'If A is ever true, B is sometimes true,' and 'B is never true.' Writing them, we have $[A\,(B)]\,(B)$. By Rule IV, from (B) we might proceed to $(B)(B)$. Hence, by Rule II (amended), from $[A\,(B)]\,(B)$ we can proceed to $[A]\,(B)$, and by Rule I to $[A]$. That is, 'A is never true.' Suppose, on the other hand, our premises are $[A\,(B)]$ and A. As before, we get $[(B)]\,A$, and by Rule III, BA, and by Rule I, B. That is, from the premises of the *modus ponens* we get the conclusion. Let us take as premises 'If A is ever true, B is sometimes true,' and 'If B is ever true, C is sometimes true.' That is, $(A\,\{B\})\,[B\,(C)]$. Then, iterating $[B\,(C)]$ within two enclosures, we get $(A\,\{B\,[B\,(C)]\})\,[B\,(C)]$, or, by Rule I, $(A\,\{B\,[B\,(C)]\})$. But we have just seen that $B\,[B\,(C)]$ can be transformed to C. Performing this under two enclosures, we get $(A\,\{C\})$, which is the conclusion, 'If A is ever true, *C* is sometimes true.' Let us now formally deduce the principle of contradiction [*A* (*A*)]. Start from any premise *X*. By Rule III we can insert [(*X*)], so that we have *X* [(*X*)]. By insertion under odd enclosures we have *X* [*A* (*X*)]. By iteration under additional enclosures we get *X* [*A* (*A X*)], by erasures under even enclosures [*A* (*A*)].

In complicated cases the multitude of enclosures become unmanageable. But by using ruled paper and drawing lines for the enclosures, composed of vertical and horizontal lines, always writing what is more enclosed lower than what is less enclosed, and what is evenly enclosed on the left-hand part of the sheet, and what is oddly enclosed on the right-hand part, this difficulty is greatly reduced. The following diagram illustrates the general style of arrangement recommended.

where unenclosed. —*A* will mean 'At some quasi-instant *A* is true.' It is equivalent to *A* simply. But -(*A*) will differ from (—*A*) or (*A*) in merely asserting that at some quasi-instant *A* is not true, instead of asserting with the latter forms that at no quasi-instant is *A* true. Our quasi-instants may be individual things. In that case —*A* will mean 'Something is *A*'; -(*A*), 'Something is not *A*'; [-(*A*)], 'Everything is *A*'; (—*A*), 'Nothing is *A*.' So *A*—*B* will express 'Some *A* is *B*'; (*A*—*B*), 'No *A* is *B*'; *A*-(*B*), 'Some *A* is not *B*'; [*A*-(*B*)], 'Whatever *A* there may be is *B*'; (*A*) (*B*), 'There is something besides *A* and *B*'; [(*A*) (*B*)], 'Everything is either *A* or *B*.' The rule of iteration must now be amended as follows.

RULE IV (amended). *Anything can be iterated under the same enclosures or under*



It is now time to make an addition to our system of symbols. Namely, *AB* signifies that *A* is at some quasi-instant true, and that *B* is at some quasi-instant true. But we wish to be able to assert that *A* and *B* are true at the same quasi-instant. We should always study to make our representations iconoidal; and a very iconoidal way of representing that there is one quasi-instant at which both *A* and *B* are true will be to connect them with a heavy line drawn in any shape, thus:

A—*B* or ┌*A*
└*B*

If this line be broken, thus *A*— —*B*, the identity ceases to be asserted. We have evidently—

RULE V. *A line of identity may be broken additional ones, its identical connections remaining identical.*

Thus, [*A*-(*B*)] can be transformed to [*A* (*A*—*B*)]. By the same rule *A*— (—*B*), i.e. 'Something is *A* and nothing is *B*,' by iteration of the line of identity, can be transformed to *A* (—*B*), i.e. 'Some *A* is not coexistent with anything that is *B*,' whence, by Rules V and II (amended), it can be further transformed to *A* (—*B*), i.e. 'Some *A* is not *B*.'

But it must be most carefully observed that two unenclosed parts cannot be illatively united by a line of identity. The enclosure of such a line is that of its least enclosed part. We can now exhibit any

ordinary syllogism. Thus, the premises of *Baroko*, 'Any *M* is *P*' and 'Some *S* is not *P*,' may be written {*M* [*P*]} *S* (*P*). Then, as just seen, we can write {*M* [*P*]} *S* (*P*). Then, by iteration, {*M* [*P* (*P*)]} *S* (*P*). Breaking the line under even enclosures, we get {[*P* (*P*)] *M*} *S* (*P*). But we have already shown that [*P* (*P*)] can be written unenclosed. Hence it can be struck out under one enclosure; and the unenclosed (*P*) can be erased. Thus we get {*M*} *S*, or 'Some *S* is not *M*.' The great number of steps into which syllogism is thus analysed shows the perfection of the method for purposes of analysis.

In taking account of relations, it is necessary to distinguish between the different sides of the letters. Thus let *l* be taken in such a sense that *X*—*l*—*Y* means '*X* loves *Y*.' Then *X* *l* *Y* will mean '*Y* loves *X*.' Then, if *m*— means 'Something is a man,' and —*w* means 'Something is a woman,' *m*—*l*—*w* will mean 'Some man loves some woman'; *m* [(*l*) *w*] will mean 'Some man loves all women'; [(*m*—*l*) *w*] will mean 'Every woman is loved by some man,' &c.

Since enclosures signify negation, by enclosing a part of the line of identity, the relation of otherness is represented. Thus, *A* ( ) *B* will assert 'Some *A* is not some *B*.' Given the premises 'Some *A* is *B*' and 'Some *C* is not *B*,' they can be written *A*—*B* *C* (*B*). By Rule III, this can be written *A* {[*B*]} *C* (*B*). By iteration, this gives *A* {[*B* (*B*)]} *C* (*B*). The lines of identity are to be conceived as passing through the space between the braces outside of the brackets. By breaking the lines under even enclosures, we get *A* {[*B* (*B*)]} *C* (*B*). As we have already seen, oddly enclosed [*B* (*B*)] can be erased. This, with erasure of the detached (*B*), gives *A* { } *C*. Joining the lines under odd enclosures, we get *A* { } *C*, or 'Some *A* is not some *C*.'

For all considerable steps in ratiocination, the reasoner has to treat qualities, or collections (they only differ grammatically), and especially relations, or systems, as objects of relation about which propositions are asserted and inferences drawn. It is, therefore, necessary to make a special study of the logical relatives '—— is a member of the collection ——,' and '—— is in the relation —— to ——.' The key to all that amounts to much in symbolical logic lies in the symbolization of these relations. But we cannot enter into this extensive subject in this article.

The system of which the slightest possible sketch has been given is not so iconoidal as the so-called Euler's diagrams; but it is by far the best *general* system which has yet been devised. The present writer has had it under examination for five years with continually increasing satisfaction. However, it is proper to notice some other systems that are now in use. Two systems which are merely extensions of Boole's algebra of logic may be mentioned. One of these is called by no more proper designation than the 'general algebra of logic.' The other is called 'Peirce's algebra of dyadic relatives.' In the former there are two operations—aggregation, which Jevons (to whom its use in algebra is due) signifies by a sign of division turned on its side, thus ·|· (I prefer to join the two dots, in order to avoid mistaking the single character for three); and composition, which is best signified by a somewhat heavy dot, •.

Thus, if *A* and *B* are propositions, *A* ·|· *B* is the proposition which is true if *A* is true, is true if *B* is true, but is such that if *A* is false and *B* is false, it is false. *A*•*B* is the proposition which is true if *A* is true and *B* is true, but is false if *A* is false and false if *B* is false. Considered from an algebraical point of view, which is the point of view of this system, these expressions *A* ·|· *B* and *A*•*B* are *mean functions*; for a mean function is defined as such a symmetrical function of several variables, that when the variables have the same value, it takes that same value. It is, therefore, wrong to consider them as addition and multiplication, unless it be that *truth* and *falsity*, the two possible states of a proposition, are considered as logarithmic infinity and zero. It is therefore well to let o represent a false proposition and ∞ (meaning logarithmic infinity, so that +∞ and −∞ are different) a true proposition. A heavy line, called an 'obelus,' over an expression negatives it.

The letters **i**, **j**, **k**, &c., written below the line after letters signifying predicates, denote individuals, or supposed individuals, of which the predicates are true. Thus, l_{ij} may mean that *i* loves *j*. To the left of the expression a series of letters Π and Σ are written, each with a special one of the individuals i, j, k attached to it in order to show in what order these individuals are to be selected, and how. Σ_i will mean that i is to be a suitably chosen individual, Π_j that j is any individual, no matter what. Thus,

$$\Sigma_i \Pi_j l_{ji}$$

means that there is an individual i such that every individual j loves i; and

$$\Pi_j \Sigma l_{ji}$$

will mean that taking any individual j, no matter what, there is some individual i, whom j loves. This is the whole of this system, which has considerable power. This use of Σ and Π was probably first introduced by O. C. Mitchell in his epoch-making paper in *Studies in Logic*, by members of the Johns Hopkins University.

In Peirce's algebra of dyadic relatives the signs of aggregation and composition are used; but it is not usual to attach indices. In place of them two relative operations are used. Let l be 'lover of,' s 'servant of.' Then ls, called the relative product of s by l, denotes 'lover of some servant of'; and $l \dagger s$, called the relative sum of l to s, denotes 'lover of whatever there may be besides servants of.' In MS. the tail of the cross will naturally be curved. The sign I is used to mean 'numerically identical with,' and T to mean 'other than.' Schröder, who has written an admirable treatise on this system (though his characters are very objectionable, and should not be used), has considerably increased its power by various devices, and especially by writing, for example, $\underset{\blacksquare}{\Pi}$ before an expression containing u to signify that u may be any relative whatever, or $\underset{\blacksquare}{\Sigma}$ to signify that it is a possible relative. In this way he introduces an abstraction or term of second intention. (C.S.P.)

Peano has made considerable use of a system of logical symbolization of his own. Mrs. Ladd-Franklin advocates eight copula-signs to begin with, in order to exhibit the equal claim to consideration of the eight propositional forms. Of these she chooses 'No a is b' and 'Some a is b' ($a \overline{\vee} b$ and $a \vee b$) as most desirable for the elements of an algorithmic scheme; they are both symmetrical and natural. She thinks that a symbolic logic which takes 'All a is b' (Boole, Schröder) as its basis is cumbrous; for every statement of a theorem, there is a corresponding statement necessary in terms of its contrapositive. This, she says, is the source of the parallel columns of theorems in Schröder's *Logik*; a single set of theorems is all-sufficient if a symmetrical pair of copulas is chosen. Some logicians (as C.S.P.) think the objections to Mrs. Ladd-Franklin's system outweigh its advantages. Other systems, as that of Wundt, show a complete misunderstanding of the problem. Cf. SYLLOGISM (2). (C.S.P., C.L.F.)

Symbolic logic finds occasion to single out two terms as of peculiar significance, and to represent them by the special symbols o (zero) and ∞ (infinity); all other terms have both application and signification, but the first of these has no object of consciousness to which it is applicable, and simply signifies the non-existent, while the second has every object of consciousness as its application, and has no signification whatever. These properties are expressed in formal language by saying that

$$a \leqslant \infty, \quad o \leqslant a$$

are, no matter what a may be, propositions of no content, though always true. But

$$\infty \leqslant a, \quad \bar{a} \leqslant o$$

state, the first, that everything is a, and the second, that $\bar{a}$ is non-existent. These last two propositions are contrapositives one of the other, and ∞ and o are a pair of contradictory terms (i. e. each is the negative of the other). Much confusion would be saved in discussions in non-symbolic logic by the recognition of these special terms. (C.L.F.)

Literature: LEIBNIZ, Philos. Schriften, t. vii, ed. Gerhardt (Berlin, 1890); Opuscules et fragments inédits de Leibniz, ed. Couturat (Paris, 1902); LAMBERT, Neues Organon (Leipzig, 1764); PLOUCQUET, Methodus calculandi in logicis (1763); GERGONNE, Essai de dialectique rationnelle, Ann. de Math., t. vii; DE MORGAN, Formal Logic (London, 1847); Syllabus of a Proposed System of Logic (1860); On the Syllogism, in Trans. Camb. Philos. Soc., viii, ix, x (1847–64); GEORGE BOOLE, The Mathematical Analysis of Logic (London, 1847); An Investigation of the Laws of Thought (Cambridge, 1854); STANLEY JEVONS, Pure Logic (London, 1864); C. S. PEIRCE, articles in Proc. Amer. Acad. Arts and Sci., vii, x, xiii; Memoirs of the same, ix, Amer. J. of Math., iii, iv, vii; ROBERT GRASSMANN, Die Begriffslehre oder Logik (Stettin, 1872); DELBŒUF, Logique algorithmique (Liège, Bruxelles, 1877); and in the Rev. Philos., ii, iii; HUGH MACCOLL, articles on the Calculus of Equivalent Statements, London Math. Soc., ix, x, xi, xvi, xxviii, xxix, xxx; and in Mind, 1880, 1897, and xi, N. S., No. 33; ERNST SCHRÖDER, Der Operationskreis des Logikkalkuls (1877); Algebra d. Logik, i (1890), ii (1) (1891); iii (1), Algebra u. Logik d. Relative

(1895); Studies in Logic, by members of the Johns Hopkins University (Peirce, Mrs. Ladd-Franklin, Mitchell, &c.) (Boston, 1883); G. PEANO, Calcolo geometrico (Turin, 1888); Arithmetices Principia, I Principii di Geometria (ibid., 1889); Formulaire de Mathématiques, en collaboration (i, 1895; ii, 1897–9; iii, 1901); and Rev. de Math., i–vii, 1891–1901); W. E. JOHNSON, The Logical Calculus, Mind, 1892; KEYNES, Studies and Exercises in Formal Logic (3rd ed., 1894); A. N. WHITEHEAD, Universal Algebra, i. Bk. II (Cambridge, 1898); EUGEN MÜLLER, Ueber d. Algebra d. Logik (Leipzig, 1900, 1901); PLATON PORETSKY, Sept lois fondamentales de la théorie des égalités logiques (Kazan, 1899); Bibliothèque du Congrès int. de Philos., iii, containing the papers of JOHNSON, MACCOLL, PORETSKY, SCHRÖDER, PEANO, BURALI-FORTI, PADOA, and PIERI (Paris, 1901). For the history of Symbolic Logic see LIARD, Les logiciens anglais contemporains (Paris, 1878); VENN, Symbolic Logic (London, 2nd ed., 1894); and PEANO's Formulaire de Mathématiques (for indications of sources of formulae). (L.C., C.L.F.)

Symbolical: Ger. *symbolisch*; Fr. *symbolique*; Ital. *simbolico*. (1) Relating to symbols in the general sense. See SYMBOL (1).

(2) Relating to symbols, novel or peculiar. In this sense the treatment of logic by means of peculiar characters or old characters put to peculiar uses is by some writers called SYMBOLIC(AL) LOGIC (q.v.).

(3) Relating to an algebraical method in which operations are denoted by letters and made the subject of operations. (C.S.P.)

Symbolism: Ger. *Symbolismus*; Fr. *symbolisme*; Ital. *simbolismo*. (1) In aesthetics: (*a*) symbols considered abstractly; (*b*) the theory of the nature and use of the SYMBOL (q. v. 2).

(2) In religion: the use of objects in a symbolic sense; that is, as sensuous emblems of spiritual acts and objects; as, for example, ritual in worship and the sacraments in one aspect of their significance.

Symbolism in this sense has a wide use in religion, the objects of which are unseen and intangible. Hence the need of helping the imagination by means of sensuous objects which may serve as fitting materializations of the spiritual. Symbolism enters into every phase of religion, including the architecture of its churches and temples. The significance of sacred architecture is never wholly that of adaptation to certain functions, but it is determined also to a degree by the spiritual import of those functions and by the influence of religious ideas. (A.T.O.)

Literature: see SYMBOL; also G. FERRERO, I simboli (1892); G. MARCHESINI, Il simbolismo (1901). (E.M.)

Symbols (and **Symbolics**) [Gr. σύμβολον, a sign]: Ger. *Symbole*; Fr. *symboles*; Ital. *simboli*. The authoritative doctrines or creeds of the Christian Church. Symbolics: a department of ecclesiastical history which treats of the origin, history, and contents of the various creeds of Christendom.

The term symbol was first employed in a theological sense by Cyprian in the year 250 A.D., and after the 4th century came into general use. It was first applied to the Apostles' Creed as a military watchword, distinguishing Christians from Pagans. Luther and Melanchthon first applied the name to Protestant confessions. Since Reformation times the use has been general.

Literature: OEHLER, Lehrb. d. Symbolik (1876); WUNDT, Symbolik d. römisch-katholischen Kirche (1880); Literature in the Creeds (1878); SCHAFF, Creeds of Christendom. (A.T.O.)

Symmetry [Gr. σύν, with, + μέτρον, measure]: Ger. *Symmetrie*: Fr. *symétrie*; Ital. *simmetria*. The arrangement in reverse order, on opposite sides of a perpendicular line or plane, of like and equal parts of an object. More loosely, the equable distribution of parts in the formation of a balanced whole.

In the latter sense it is almost synonymous with proportion, consistency, and congruity. In the narrower sense applied most appropriately in architecture and sculpture; more ambiguously in drawing and painting. Applied rarely and somewhat metaphorically to canon and fugue in music, referring to the temporal repetition of musically similar passages, to metrical relations, as in the asclepiadic verse, and to the structure of the drama, as involving 'exposition,' 'conflict,' and 'solution.' For closely connected meanings see BALANCE, HARMONY, and PROPORTION.

The Greek term was probably first applied to the commensurability of numbers, thence to the parts of a statue, and finally to the relations of form in general. The aesthetic value of the quality has been recognized by practically all aestheticians from the earliest Greek writers down to the present day. The principle, with its connected categories, harmony and proportion, is, however, so fundamental to the Greek conception of beauty, that it plays relatively a more important