"There is continual spring, and harvest there
Continual, both meeting at one time:
For both the boughs do laughing blossoms bear
And with fresh colours deck the wanton prime,
And eke at once the heavy trees they climb,
Which seem to labour under their fruit's load."

ALFRED WAITES.

WORCESTER, MASS., October 18, 1903.

PRACTICAL APPLICATION OF THE THEORY OF FUNCTIONS.

TO THE EDITOR OF THE NATION:

SIR: In your interesting editorial of October 1, you say, concerning the theory of numbers and the theory of functions, that "no *practical* application, and (as far as one can see into the future) no *scientific* application, either, is likely ever to be made of one or other of those two theories, outside of pure mathematics itself." This statement would probably have passed unchallenged a few years ago, perhaps even as recently as the time when the lamented mathematician to whom you allude was devoting himself to that pure and secluded virgin, the theory of numbers; but the mathematicians and physicists have been coming together within the last decade; they are comparing their standing problems, and are learning how to be mutually helpful and stimulating. It is realized that certain physical problems are at a standstill for want of appropriate modes of mathematical expression, and that there is need for the invention or discovery of new forms of functional relationship. Increased attention is accordingly being directed to the wide field of the theory of functions of a complex variable. The complex plane is one of the meeting-grounds of mathematicians and physicists, and the latter are now quite at home in the presence of that coy handmaiden, the complex variable; indeed, the well-known transformation scene in which she and her image play such a prominent part, is now an important feature in the solution of some practical problems.

The discovery of a new form of function to correspond to a new physical relation is at once a gain to natural science and to mathematics, and it widens the intellectual horizon. A single concrete example of the way in which the mutual stimulus operates may be of interest.

In 1891 Dr. Pockels of Heidelberg, in his treatise on the partial differential equations of mathematical physics, made the following suggestive remarks:

"Both from a mathematical and physical standpoint, multiform functions are important, and it is very desirable that the properties of such functions, their winding-points and singularities, their behavior on Riemann surfaces, etc., should be systematically investigated—in short, all the function-theory questions which were handled in the theory of the Newtonian and logarithmic potential. . . . In this direction of inquiry without doubt a wide and rich field offers itself."

About six years later, Professor Sommerfeld of Aachen, and his pupil, Dr. Carslaw of Glasgow, contributed papers to the London Mathematical Society on the multiform functions that satisfy certain differential equations subject to various boundary conditions; and they used these functions to solve some standing physical problems. One of these was the following problem in diffraction: "Plane waves of sound, light, or electricity are incident on a thin infinite half-plane bounded by a straight edge: to find the resulting diffraction of the waves." This problem had been mentioned in Lord Rayleigh's article on 'Wave Theory' in the 'Encyclopædia Britannica,' in the following terms:

"The full solution of problems concerning the mode of action of a screen is scarcely to be expected. Even in the simple case of sound, where we know what we have to deal with, the mathematical difficulties are formidable, and we are not able to solve such an apparently elementary question as the transmission of sound past a rigid infinite thin plane screen bounded by a straight edge or perforated by a circular aperture."

The appropriate solution was obtained by the conception of a twofold Riemann space having the ordinary physical space as one of its folds, the half-plane as a branch membrane, and the straight edge as a winding line. The characteristic multiform function that expresses the wave motion was derived by the most beautiful use of pure function-theory.

I doubt if either Cayley or Sylvester would regret to see the sway of the virgin queen thus extended over new dominions. If she no longer has the seclusion of a "Leibnitzian monad," she exemplifies in a higher sense the Leibnitzian doctrine of preëstablished harmony—the harmony of the world of nature with the world of the intellect. What God hath joined together, let not man put asunder.

Very truly yours, JAMES MCMAHON.

ITHACA, N. Y., October 10, 1903.

[An apposite instance! Had we known of it, we should have softened our remark. Nor, in making it, did we forget that several applications have been made of propositions worked out by Cauchy and earlier mathematicians before the "theory of functions" was christened, and for which (though they are now incorporated in it) it can take no more credit than the theory of numbers could to the carpenter's rule of three-four-five, which was known to Pythagoras, but the principle of which may to-day figure in a treatise on that theory. But now, for the first time, we meet with one genuine case of an application of the theory of functions, upon which we may ground some hope for further such triumphs. It is a convincing and striking proof that a line of thought which seems to relate exclusively to impossible states of things may, if resolutely pursued, eventually bring great light upon familiar experiences. Let us give this instance a permanent place in our memory alongside of the fact that Pascal, after his wonderful discovery about conic sections, abandoned that study as an idle pastime having no application to any matter of importance.

All this, however, does not in the least touch the point that our remark was designed to make; for at the time when it could be said that the British were neglecting the theory of functions (which is no longer true), there was no glimmer of reasonable hope that it could ever be of any use. As for the theory of numbers, the first application of it has, we believe, yet to be made, unless, perhaps, Cayley used it for his theory of chemical "trees." But that concerns the *partition* of numbers, a separate branch of mathematics which the English have perfected, we believe.

We are heartily of opinion—but it is no longer a matter of opinion—that the younger generation of physicists are going to reap a rich harvest from their studies of the higher mathematics. In this they are only following a time-honored custom, for almost all the great physicists, from Galileo down, have been strong mathematicians. At the same time, there are instances enough—like the beautiful researches of Le Bon on phosphorescence and peculiar radiations—to show that, even in these days, the consciousness of a decidedly deficient capacity for mathematics need discourage no young man, nor young woman, from devoting himself or herself to physical investigations.—ED. NATION.]

Notes.

A new edition of Stevenson's 'Dr. Jekyll and Mr. Hyde,' with illustrations by Charles Raymond Macauley, and a reissue of the century-old 'Peter Piper's Principles of Polite Pronunciation' (for children), may be looked for next month from the Scott-Thaw Co. of this city.

The pioneer missionary of the Episcopal Church in the Northwest, Dr. J. Lloyd Breck, is to be commemorated in a volume of biography and reminiscences by the Rev. Theodore I. Holcomb published by Thomas Whittaker, under the title 'An Apostle of the Wilderness.'

L. C. Page & Co.'s fall announcements include 'Belgium: Its Cities,' by Grant Allen; 'Gardens of the Caribbees,' by Ida M. H. Starr; 'The Art of the Pitti Palace,' by Julia de W. Addison; 'The Cathedrals of Northern France,' by Francis Miltoun; 'Japanese Art,' by Sadakichi Hartmann; 'The Love Affairs of Great Musicians,' by Rupert Hughes; 'Stevenson's Shrine: A Record of a Pilgrimage,' by Laura Stubbs; 'The Kinship of Nature,' essays by Bliss Carman, with 'Sappho: One Hundred Lyrics,' by the same writer; and 'Militarism: or, Peace and War at the Beginning of the 20th Century,' by G. Ferrero.

'Two Argonauts in Spain,' Jerome Hart's letters to the San Francisco *Argonaut*, of which he is editor, will be brought out by the Argonaut Co.

We spoke recently of the approaching completion of the A. L. A. Index to Portraits. This great undertaking is, we learn from the September Bulletin of the Central National Library in Florence, to be paralleled (and of course overlapped) by that institution's own 'Indice dei Ritratti,' which has been in compilation for a decade. The impulse to this index was the gift to the Library of more than 20,000 portraits by Professor Buonamici; but the range of the work transcends this collection to embrace other portraits possessed by the Library as well as those in books. In this last particular, a base exists in a manuscript 'Iconografia Universale' begun in 1828 by Vincenzo Follini, in sixteen vol-