

"There is continual spring, and harvest there
Continual, both meeting at one time:
For both the boughs do laughing blossoms bear
And with fresh colours deck the wanton prime,
And eke at once the heavy trees they climb,
Which seem to labour under their fruit's load."

ALFRED WAITES.

WORCESTER, MASS., October 18, 1903.

PRACTICAL APPLICATION OF THE THEORY OF FUNCTIONS.

TO THE EDITOR OF THE NATION:

SIR: In your interesting editorial of October 1, you say, concerning the theory of numbers and the theory of functions, that "no practical application, and (as far as one can see into the future) no scientific application, either, is likely ever to be made of one or other of those two theories, outside of pure mathematics itself." This statement would probably have passed unchallenged a few years ago, perhaps even as recently as the time when the lamented mathematician to whom you allude was devoting himself to that pure and secluded virgin, the theory of numbers; but the mathematicians and physicists have been coming together within the last decade; they are comparing their standing problems, and are learning how to be mutually helpful and stimulating. It is realized that certain physical problems are at a standstill for want of appropriate modes of mathematical expression, and that there is need for the invention or discovery of new forms of functional relationship. Increased attention is accordingly being directed to the wide field of the theory of functions of a complex variable. The complex plane is one of the meeting-grounds of mathematicians and physicists, and the latter are now quite at home in the presence of that coy handmaiden, the complex variable; indeed, the well-known transformation scene in which she and her image play such a prominent part, is now an important feature in the solution of some practical problems.

The discovery of a new form of function to correspond to a new physical relation is at once a gain to natural science and to mathematics, and it widens the intellectual horizon. A single concrete example of the way in which the mutual stimulus operates may be of interest.

In 1891 Dr. Pockels of Heidelberg, in his treatise on the partial differential equations of mathematical physics, made the following suggestive remarks:

"Both from a mathematical and physical standpoint, multiform functions are important, and it is very desirable that the properties of such functions, their winding-points and singularities, their behavior on Riemann surfaces, etc., should be systematically investigated—in short, all the function-theory questions which were handled in the theory of the Newtonian and logarithmic potential. . . . In this direction of inquiry without doubt a wide and rich field offers itself."

About six years later, Professor Sommerfeld of Aachen, and his pupil, Dr. Carslaw of Glasgow, contributed papers to the London Mathematical Society on the multiform functions that satisfy certain differential equations subject to various boundary conditions; and they used these functions to solve some standing physical problems. One of these was the following problem in diffraction: "Plane waves of sound, light, or electricity are incident on a thin infinite half-plane bounded by a straight edge: to find the resulting diffraction of the

waves." This problem had been mentioned in Lord Rayleigh's article on 'Wave Theory' in the 'Encyclopædia Britannica,' in the following terms:

"The full solution of problems concerning the mode of action of a screen is scarcely to be expected. Even in the simple case of sound, where we know what we have to deal with, the mathematical difficulties are formidable, and we are not able to solve such an apparently elementary question as the transmission of sound past a rigid infinite thin plane screen bounded by a straight edge of perforated by a circular aperture."

The appropriate solution was obtained by the conception of a twofold Riemann space having the ordinary physical space as one of its folds, the half-plane as a branch membrane, and the straight edge as a winding line. The characteristic multiform function that expresses the wave motion was derived by the most beautiful use of pure function-theory.

I doubt if either Cayley or Sylvester would regret to see the sway of the virgin queen thus extended over new dominions. If she no longer has the seclusion of a "Leibnizian monad," she exemplifies in a higher sense the Leibnizian doctrine of preestablished harmony—the harmony of the world of nature with the world of the intellect. What God hath joined together, let not man put asunder.

Very truly yours, JAMES McMAHON.

ITHACA, N. Y., October 10, 1903.

[An apposite instance! Had we known of it, we should have softened our remark. Nor, in making it, did we forget that several applications have been made of propositions worked out by Cauchy and earlier mathematicians before the "theory of functions" was christened, and for which (though they are now incorporated in it) it can take no more credit than the theory of numbers could to the carpenter's rule of three-four-five, which was known to Pythagoras, but the principle of which may to-day figure in a treatise on that theory. But now, for the first time, we meet with one genuine case of an application of the theory of functions, upon which we may ground some hope for further such triumphs. It is a convincing and striking proof that a line of thought which seems to relate exclusively to impossible states of things may, if resolutely pursued, eventually bring great light upon familiar experiences. Let us give this instance a permanent place in our memory alongside of the fact that Pascal, after his wonderful discovery about conic sections, abandoned that study as an idle pastime having no application to any matter of importance.

All this, however, does not in the least touch the point that our remark was designed to make; for at the time when it could be said that the British were neglecting the theory of functions (which is no longer true), there was no glimmer of reasonable hope that it could ever be of any use. As for the theory of numbers, the first application of it has, we believe, yet to be made,

unless, perhaps, Cayley used it for his theory of chemical "trees." But that concerns the *partition* of numbers, a separate branch of mathematics which the English have perfected, we believe.

We are heartily of opinion—but it is no longer a matter of opinion—that the younger generation of physicists are going to reap a rich harvest from their studies of the higher mathematics. In this they are only following a time-honored custom, for almost all the great physicists, from Galileo down, have been strong mathematicians. At the same time, there are instances enough—like the beautiful researches of Le Bon on phosphorescence and peculiar radiations—to show that, even in these days, the consciousness of a decidedly deficient capacity for mathematics need discourage no young man, nor young woman, from devoting himself or herself to physical investigations.—ED. NATION.]

Notes.

A new edition of Stevenson's 'Dr. Jekyll and Mr. Hyde,' with illustrations by Charles Raymond Macaulay, and a reissue of the century-old 'Peter Piper's Principles of Polite Pronunciation' (for children), may be looked for next month from the Scott-Thaw Co. of this city.

The pioneer missionary of the Episcopal Church in the Northwest, Dr. J. Lloyd Breck, is to be commemorated in a volume of biography and reminiscences by the Rev. Theodore I. Holcomb published by Thomas Whittaker, under the title 'An Apostle of the Wilderness.'

L. C. Page & Co.'s fall announcements include 'Belgium: Its Cities,' by Grant Allen; 'Gardens of the Caribbees,' by Ida M. H. Starr; 'The Art of the Pitti Palace,' by Julia de W. Addison; 'The Cathedrals of Northern France,' by Francis Miltoun; 'Japanese Art,' by Sadakichi Hartmann; 'The Love Affairs of Great Musicians,' by Rupert Hughes; 'Stevenson's Shrine: A Record of a Pilgrimage,' by Laura Stubbs; 'The Kinship of Nature,' essays by Bliss Carman, with 'Sappho: One Hundred Lyrics,' by the same writer; and 'Militarism: or, Peace and War at the Beginning of the 20th Century,' by G. Ferrero.

'Two Argonauts in Spain,' Jerome Hart's letters to the San Francisco *Argonaut*, of which he is editor, will be brought out by the Argonaut Co.

We spoke recently of the approaching completion of the A. L. A. Index to Portraits. This great undertaking is, we learn from the September Bulletin of the Central National Library in Florence, to be paralleled (and of course overlapped) by that institution's own 'Indice dei Ritratti,' which has been in compilation for a decade. The impulse to this index was the gift to the Library of more than 20,000 portraits by Professor Buonamici; but the range of the work transcends this collection to embrace other portraits possessed by the Library as well as those in books. In this last particular, a base exists in a manuscript 'Iconografia Universale' begun in 1828 by Vincenzo Follini, in sixteen vol-

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variously explain'd; is mention'd by Theophrastus, Aristotle, Plato, Pausanias, Athenaeus, Eustathius, and a Croud of Authors more, who are quoted in *Castellanus*, and *Meursius* in his *Græcia Fertata*." In a later edition, Theobald quoted the 'Faerie Queene.' In 1744 Hanmer referred to Erasmus, while in 1747 Warburton summed up what had been said on the subject by Theobald, Hanmer, Bentley, and Pearce (the last two in their notes on 'Paradise Lost,' ix. 440).

What, then, has been well known to others for a hundred and seventy years is still unknown to certain Baconians. But this is not all. Even if Shakspeare was the author of "Henry VI.," even if he could have obtained his knowledge of the gardens of Adonis only from Plato, not translated into English in his day—still, Lord Penzance's conclusion that Shakspeare must have "mastered the Greek language" by no means follows. Lord Penzance does not seem to have been aware of the fact that in the sixteenth century it was common to print the works of Greek authors either in a Greek text, or in a Greek and Latin text, or in a Latin text only. There are now before me a Latin text of Plato printed in 1561, and a Greek and Latin text of Plutarch printed in 1572. It is thus seen that Shakspeare could have leaped of the gardens of Adonis through the Latin of Plato, Pliny, and Plutarch, and of the Adonia through the Latin of Plutarch and the English of Sir Thomas North. Hence the "rather startling result" which so puzzled Lord Penzance, and which he said demanded an explanation, is, like so many other mare's-nests discovered by the Baconians, purely a difficulty of their own creating.

ALBERT MATTHEWS.

Boston, October 26, 1903.

P. S.—It may be added that the gardens of Adonis were again alluded to by Spenser in 'Colin Clout,' l. 804, and were mentioned by Jonson in 1599 ('Every Man out of his Humour,' Act iv., Sc. viii.).

FRANCIS ELLINGWOOD ABBOT.

TO THE EDITOR OF THE NATION:

SIR: Dr. Francis Ellingwood Abbot was found, in Cambridge, on October 23, lying face downward on the grave of his wife, who had died just ten years ago, he having lately completed the book on philosophy to which he had devoted his whole life. One of the benefits of an acquaintance with Dr. Abbot was that it gave a new conception of the saying, "The pure in heart shall see God." The unsophisticated purity of his love of and apprehension of truth, oblivious of the tide of opinion, was a quality without which the introduction to his 'Scientific Theism,' wherein he put his finger unerringly (as the present writer thinks) upon the one great blunder of all modern philosophy, could not have been written. The perfect clearness and simplicity of his argument will blind many a mind to it that could thread its way through the most abstruse tortuosities of law. But Dr. Abbot was like that "best philosopher" of whom Wordsworth speaks—

"On whom those truths do rest
Which we are tolling all our lives to find."

He had that spiritual insight into philosophy that Wordsworth attributes to the child. In each writing of a philosophical

nature that he produced, he brought out some undeniable and important point that had been almost entirely overlooked by philosophers; so that philosophy is sure to be materially advanced by the great work which he left finished, but which is not yet published.

Dr. Abbot was an intensely self-conscious man, with a perfectly accurate appreciation of himself, and with a noble variety of self-consciousness that bent to no compromise with himself or with others, for all his gentle and loving nature. The world does not like men so extremely consistent as he was, but it seems impossible at this day to review any long-past episode of Dr. Abbot's life without seeing that his course was the right one, the only right one. He was fond of writing verses. Here are two stanzas of seven in his 'Scientific Theism':

"Art Thou the Truth?
To Thee, then, loved and craved and sought of yore,
I consecrate my manhood o'er and o'er,
As erst my youth."

"Art Thou the Strong?
To Thee, then, though the air be thick with night,
I trust the seeming-unsafe Right,
And leave the wrong."

October 27, 1903.

C. S. P.

Notes.

The Burrows Brothers' Company of Cleveland project a further extension of their reprints of scarce and neglected Americana, in limited editions, with Elliot's 'Indian Logic Primer,' Hutchins's 'Topographical Description,' Leonard's 'Narratives of Adventures,' and 'Narratives of Indian Captivities.' As heretofore, competent editors are provided in each case.

Harper's November list includes 'America in Literature,' by Prof. George E. Woodberry; 'The Standard of English Pronunciation,' by Prof. T. R. Lounsbury; 'The Making of a Journalist,' by Julian Ralph; 'Tennyson's Suppressed Poems'; 'The Ambassadors,' by Henry James; 'The Russian Advance,' by Albert J. Beveridge; 'The Keystone of Empire'; 'Symbol-Psychology: A New Interpretation of Race Traditions,' by the Rev. Adolph Roeder; 'Winter,' photographically illustrated, by Rudolph Eickemeyer, Jr.; William Morris's 'Pygmalion and the Image,' illustrated; and 'Mother and Father,' by Rob Rolfe Gilson, illustrated by Alice Barber Stephens.

Ginn & Co. announce 'The Louisiana Purchase,' by Ripley Hitchcock; 'Greek Sculpture: Its Spirit and Principles,' by Edmund von Mach; and 'Bacteria, Yeasts and Molds in the Home,' by Prof. H. W. Conn.

'Food and Cookery for the Sick and Convalescent,' by Miss Fannie Merritt Palmer, is in the press of Little, Brown & Co.

A. Weasels Co. will shortly publish 'Famous Battles of the Nineteenth Century,' and 'Napoleon Bonaparte,' for boys, by William C. Sprague.

E. P. Dutton & Co. will soon bring out 'Plays, Acting and Music,' by Arthur Symonds.

Nearly ready is Esther Singleton's 'French and English Literature,' from McClure, Phillips & Co.

The history of New Hampshire is to be told by several writers in a volume edited by George Franklyn Willey and issued by the New Hampshire Publishing Corporation under the title of 'State Builders.'

Whether it be or be not commendable to collect and publish every discoverable scrap from the pen of a departed man of letters—little ephemeral squibs in newspapers, album-verses, and miscellaneous *quisquillia*—is a question too ample to be discussed in the compass of a note, but it is suggested by the appearance of Volumes I. and V. of the 'Works of Charles and Mary Lamb,' edited by E. V. Lucas (London: Methuen & Co.; New York: G. P. Putnam's Sons). Certainly, if such republication be ever justified, it should be so in the case of Lamb, everything from whose pen has some flavor of his quaintness and whimsicality. While the present edition includes all his work previously published, the editor has spared no pains in tracking his author (often by scent rather than by sight) through files of forgotten journals and other dust-heaps of literature, and has perhaps given even more labor to the elucidation of obscure allusions to persons and happenings of the time, embodying the results of these researches in a mass of notes. The inclusion of Mary Lamb's writings gives this edition a special value, and two well-engraved portraits are an acceptable adornment of these handsome volumes.

With volume seven the sumptuous Macmillan edition of the 'Letters and Literary Remains of Edward FitzGerald' is brought to a close—deliberate, we might call it, in view of the protracted publication covering many months. The miscellaneous character of the present volume, which begins with Omar in all his glory of four versions, and all but ends with the Chronology of Charles Lamb, reflects many phases of FitzGerald. Here are translations from the Persian and from Petrarch; his own short flights of poetry; a letter to Professor Cowell; his preface to 'Polonius' and introduction to 'Readings from Crabbe'; and the admirably prosed obituary notices of Crabbe and of Bernard Barton. His Greek and Spanish sides are indeed wanting, and still the medley contains all of the man if not of the poet. Those who can afford this edition will seek it; it represents, we think, the last word unless the Letters should be reprinted by themselves, as they deserve to be, being for the first time arranged chronologically.

One of the happiest selections for the Miniature Temple Classics was Sebastian Evans's translation of 'The High History of the Holy Grail.' It has seemed good to the publishers, Dent-Dutton, to offer his excellent version in more stately form. The result is an immaculate reprint in octavo, in which nothing but the illustrations may be cavilled at. Jessie M. King has unhappily taken as her exemplar the morbid mysticism of Aubrey Beardsley in his illustrations for the 'Morte d'Arthur' but she has little of his perverse ability in the management of the pounced line and the solid spot.

Three editions of W. H. Hudson's 'The Naturalist in La Plata' followed rapidly upon its first appearance in 1872, when we judged it a work of rare originality and authority, delightful to read. The demand, it appears, has continued unsatisfied, and now the treatise (for it is not quite a narrative) is once more handsomely produced, with some changes in the illustrations, by J. M. Dent & Co., London (New York: Dutton).

Henry Harland's sprightly story of 'The Cardinal's Snuff-box' has been freshly brought out by John Lane in very attractive

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