THE MONIST

A QUARTERLY MAGAZINE

DEVOTED TO THE PHILOSOPHY OF SCIENCE

VOLUME XIX.

CHICAGO
THE OPEN COURT PUBLISHING COMPANY
1909

Reprinted with the permission of the original publishers

KRAUS REPRINT CORPORATION
New York
1966

SOME AMAZING MAZES.

A SECOND CURIOSITY.

A phenomenon easier to understand depends on the fact that, in counting round and round, a cycle of 53 numbers, $\sqrt{-1} = \pm 30$. (For $30^2 = 900 = 17 \cdot 53 - 1$.) This, likewise, may be exhibited in the form of a "trick." You begin with a pack of 52 playing-cards arranged in regular order. For this purpose, it is necessary to assign ordinal numbers to the four suits. It seems appropriate to number the spade-suit as 1, because its ace carries the maker's trade-mark. I would number the heart-suit 2, because the pips are partially cleft in two; the club-suit 3, because a "club," as the French term *trèfle* reminds us, is a trefoil; and the diamond-suit as 4 or 0, because the pips are quadrilaterals, and counting round and round a cycle of 4, $4 = 0$. But it is convenient, in numbering the cards, to employ the system of arithmetical notation whose base is 13. It will follow that, if the cards of each suit are to follow the order 1 2 3 4 5 6 7 8 9 X J Q K, the king of each suit must be numbered as if it were a zero-card of the following suit. The inconvenience of this is very trifling compared with the convenience of directly availing oneself of a regular system of notation; for the exhibitor of the "trick" will have many a "long multiplication" to perform in his head, as will shortly appear. Another slight inconvenience is that the cycle of numeration must be fifty-three, or 4♠, which, or its highest possible multiple, must be sub-

tracted from every product that exceeds 4♠. It is to be remembered that ♢, ♠, ♡, ♣, are used as nothing but other shaped characters for 0, 1, 2, 3, respectively. Thirteen is the base of numeration, but fifty-three, or 4♠, is the cycle of numeration. I adopt ♢, rather then K, as the zero-sign in order to avoid denoting the king of diamonds by ♠ K, etc. In order to exhibit the trick in the highest style, the performer should have this multiplication table

♢ ♠	♢ ♡	♢ ♣	♢ 4	♢ 5	♢ 6	♢ 7	♢ 8	♢ 9	♢ 10	♢ J	♢ Q
♢ ♡	♢ 4	♢ 6	♢ 8	♢ 10	♢ Q	♠ ♠	♠ ♣	♠ 5	♠ 7	♠ 9	♠ J
♢ ♣	♢ 6	♢ 9	♢ Q	♠ ♡	♠ 5	♠ 8	♠ J	♡ ♠	♡ 4	♡ 7	♡ 10
♢ 4	♢ 8	♢ Q	♠ ♣	♠ 7	♠ J	♡ ♡	♡ 6	♡ 10	♣ ♠	♣ 5	♣ 9
♢ 5	♢ 10	♠ ♡	♠ 7	♠ Q	♡ 4	♡ 9	♣ ♠	♣ 6	♣ J	4 ♣ ♢ ♡	4 8 ♢ 7
♢ 6	♢ Q	♠ 5	♠ J	♡ 4	♡ 10	♣ 3	♣ 9	4 ♡ ♢ ♠	4 8 ♢ 7	5 ♠ ♠ ♢	5 7 ♠ 6
♢ 7	♠ ♠	♠ 8	♡ ♡	♡ 9	♣ ♣	♣ 10	4 4 ♢ ♣	4 J ♢ 10	5 5 ♠ 4	5 Q ♠ J	6 6 ♡ 5
♢ 8	♠ ♣	♠ J	♡ 6	♣ ♠	♣ 9	4 4 ♢ ♣	4 Q ♢ J	5 7 ♠ 6	6 ♡ ♡ ♠	6 10 ♡ 9	7 5 ♣ 4
♢ 9	♠ 5	♡ ♠	♡ 10	♣ 6	4 ♡ ♢ ♠	4 J ♢ 10	5 7 ♠ 6	6 ♣ ♡ ♡	6 Q ♡ J	7 8 ♣ 7	8 4 ♢ ♡
♢ 10	♠ 7	♡ 4	♣ ♠	♣ J	4 8 ♢ 7	5 5 ♠ 4	6 ♡ ♡ ♠	6 Q ♡ J	7 9 ♣ 8	8 6 ♢ 4	9 ♣ ♠ ♠
♢ J	♠ 9	♡ 7	♣ 5	4 ♣ ♢ ♡	5 ♠ ♠ ♢	5 Q ♠ J	6 10 ♡ 9	7 8 ♣ 7	8 6 ♢ 4	9 4 ♠ ♡	10 ♡ ♡ ♢
♢ Q	♠ J	♡ 10	♣ 9	4 8 ♢ 7	5 7 ♠ 6	6 6 ♡ 5	7 5 ♣ 4	8 4 ♢ ♡	9 ♣ ♠ ♠	10 ♡ ♡ ♢	J ♠ ♡ Q

by heart in which I have been forced to put 10 in place of X most incongruously simply because I am informed that the latter would transcend the resources of the printing-office.

Yet I do it quite passably without possessing that accomplishment. In those squares of the multiplication-table where two lines are occupied, the upper gives the simple product in tridecimal notation, and the lower the remainder of this after subtracting the highest less multiple of fifty-three, i. e., of 4♠.

In order to exhibit the trick, while you are arranging the cards in regular order, you may tell some anecdote which involves some mention of the numbers 5 and 6. For instance, you may illustrate the natural inaptitude of the human animal for mathematics, by saying how all peoples use some multiple of 5 as the base of numeration, because they have 5 fingers on a hand, although any person with any turn for mathematics would see that it would be much simpler, in counting on the fingers, to use 6 as the base of numeration. For having counted 5 on the fingers of one hand, one would simply fold a finger of the other hand for 6, and then make the first finger of the first hand to continue the count. The object of telling this anecdote would be to cause the numbers 5 and 6 to be uppermost in the minds of the company. But you must be very careful not at all to emphasize them; for if you do, you will cause their avoidance. The pack being arranged in regular sequence, you ask the company into how many piles you shall deal them, and if anybody says 5 or 6, deal into that number of piles. If they give some other number, manifest not the slightest shade of preference for one number of piles over another; but have the cards dealt again and again, until you can get for the last card either ♠x, that is, the ten of the second suit, (i. e., suit number one; since the first suit is numbered ◇, or zero), or ♡4, the four of the third suit, or ♠6, or ♡8. If you cannot influence the company to give you any of the right numbers, after they have ordered several deals, you can say, "Now let me choose a couple of numbers," and by looking through the pack, you will probably find that one or other of those can be brought to the face of the pack in two or three deals. For every deal multiplies the ordinal place of each card by a certain number, counting round and round a cycle of 53. And this multiplier is that number which multiplied by the number of piles in the deal gives +1 or −1 in counting round and round the cycle of 53. For it makes no difference to which end of the pack the card is drawn. After each deal the piles are to be gathered up according to the same rule as in the first "trick," except that the first pile taken must not be the one on which the 52nd card fell, but the one on which the 53rd would have fallen if there had been 53 cards in the pile. The last deal having been made, you lay all the cards now, backs up, in 4 rows of 13 cards in each row, leaving small gaps between the 3rd and 4th and 6th and 7th cards counting from each end, thus:

1	2	3	4	5	6	7	8	9	10	J	Q	K
K	Q	J	10	9	8	7	6	5	4	3	2	1

The object of these gaps is to facilitate the counting of the places from each end, both by yourself and by the company of onlookers. If the first or last card is either ♠x or ♡4, the first card of the pack will form the left-hand end of the top row, and each successive card will be next to the right of the previously laid card, until you come to the end of a row, when the next card will be the extreme left-hand card of the row next below that last formed. But if the first or last card is either ♠6 or ♡8, you begin at the top of the extreme right-hand column, and lay down the following three cards each under the last, the fifth card forming the head of the column next to the left, and so on, the cards being laid down in successive columns, passing downward in each column, and the successive columns toward the right being formed in regular order.

You now explain to the company, very fully and clearly, that the upper row consists of the places of the diamonds; and you count the places, pointing to each, thus: "Ace of diamonds, two of diamonds, three; four, five, six; the seven, a little separated, the eight, nine, and ten, together; then a little gap, and the knave, queen, king of diamonds together. The next row is for the spades in the same regular order, from that end to this," (you will not say "right" and "left," because the spectators will probably be at different sides of the table,) "next the hearts, and last the clubs. Please remember the order of the suits, diamond," (you sweep your finger over the different rows successively) "spades, hearts, and clubs. But," you continue, "those are the places beginning at *that*" (the upper left-hand) "corner. In addition, every card has a *second* place, beginning at *this* opposite corner," (the lower right-hand corner.) "The order is the same; only you count backwards, toward the right in each row; and the order of the suits is the same, diamonds, spades, hearts, clubs; only the places of the diamonds are in the bottom row, the places of the spades next above them, the places of the hearts next above them, and the clubs at the top. These are the regular places for the cards. But owing to their having been dealt out so many times, they are now, of course, all out of both their places." You now request one of the company (not the least intelligent of them,) simply to turn over any card in its place. Suppose he turns up the fifth card in the third row. It will be either the ♡ 3 or ♠ J. Suppose it is the former. Then you say, "Since the three of hearts is in the place of the five of hearts, counting from *that* corner, it follows *of course*" (don't omit this phrase, nor emphasize [illegible]; but say it as if what follows were quite a syllogisti[illegible] evident conclusion,) "that the five of hearts will be in the place of the three of hearts counting from the opposite corner." Thereupon, you count "Spades, hearts: one, two, three," and turn up the card, which, sure enough, will be ♡ 5. "But," you continue, "counting from the first corner, the five of hearts is in the place of the knave of spades, and accordingly, the

knave of spades will, of course, be in the place of the five of hearts, counting from the opposite corner." You count, first, to show that ♡ 5 is in the place of ♠ J, and then, always pointing as you count, and counting, first the rows, by giving successively the names of the suits, "diamonds, spades, hearts," and then the places in the row, "one, two, three, four, five," and turning up the card you find it to be, as predicted, the ♠ J. "Now," you continue, "the knave of spades is in the place of the nine of spades counting from the first corner, so that we shall necessarily find the nine of spades in the place of the knave of spades counting from the opposite corner." You count as before, and find your prediction verified. [I will here interrupt the description of the "trick" to remark that the number of different arrangements of the fifty-two cards all possessing this same property is thirty-eight thousand three hundred and eighty-two billions (or millions squared), three hundred and seventy-six thousand two hundred and sixty-six millions, two hundred and forty thousand, $= 6 \times 10 \times 14 \times 18 \times 22 \times 26 \times 30 \times 34 \times 38 \times 42 \times 46 \times 50$, not counting a turning over of the block as altering the arrangement. But of these only one arrangement can be produced by dealing the cards according to our general rule. Either of the four *simplest* arrangements having the property in question will be obtained by first laying out the diamonds in a row so that the values of the cards increase regularly in passing along the row in either direction, then laying out the spades in a parallel row either above or below the diamonds, but leaving space for another row between the diamonds and spades, their values increasing in the counter-direction to the diamonds, then laying out the hearts in a parallel row close upon the other side of the diamonds, their values increasing in the same direction as the spades, and finally laying out the clubs between the diamond-row and the spade-row, their values increasing in the same direction as the former.

Not to let slip an opportunity for a logical remark, let me note that, *in itself considered*, i. e., regardless of their sequence of values, any one arrangement of the cards is as *simple* as any other; just as any continuous line that returns into itself, without crossing or touching itself, or branching, is just as simple, *in itself*, as any other; and relatively to the sequence of values of the cards, only, the arrangement produced in "trick," in which the value of each card is i times the ordinal number of its place, where $i = \pm\sqrt{-1}$, is far simpler than the arrangement just described. But in calling the latter arrangement the "simpler," I use this word in the sense that is most important in logical methodeutic; namely, to mean more facile of human imagination. We form a detailed icon of it in our minds more readily.]

You now promptly turn down again the four cards that have been turned up (for some of the company may have the impression that the proceeding might continue indefinitely; and you do not wish to shatter their pleasing illusions,) and ask how many piles they would like to have the cards dealt in next. If they mention 5 or 6, you say, "Well we will deal them into 5 and 6. Or shall we deal them into 4, 5, 6? Or into 2 and 7? Take your choice." Which ever they choose, you say, "Now in what order shall I make the dealings?" It makes no difference. But how the cards are to be taken up will be described below. After gathering the cards in the mode described in the next paragraph, deal them out, *without turning the cards up*. [I have never tried what I am now describing; but for fear of error, I shall do so before my article goes to press.] After that, you say, "Oh, I don't believe they are sufficiently shuffled. I will milk them." You proceed to do so. That is, holding the pack backs up, you take off the

cards now at the top and bottom, and lay them backs up, the card from the bottom remaining at the bottom; and this you repeat 25 times more, thus exhausting the pack. Many persons insist that the proper way of milking the cards is to begin by putting the card that is at the back of the pack at its face; but when I speak of "milking," I mean this *not* to be done. Having milked the pack three times, you count off the four top cards (i. e., the cards that are at the top as you hold the pack with the faces down,) one by one from one hand to the other, putting each card above the last, so as to reverse their positions. You then count the next four into the same receiving hand, *under* the four just taken, so that their relative positions remain the same. The next four are to be counted, one by one, upon the first four, so that their relative positions are reversed, and the next four are to be counted into the receiving hand under those it already holds. So you proceed alternately counting four to the top and four to the bottom of those already in the receiving hand, until the pack is exhausted. You then say, "Now we will play a hand of whist." You allow somebody to cut the cards and deal the pack, as in whist, one by one into four "hands," or packets, turning up the last card for the trump. It will be found that you hold all the trumps, and each of the other players the whole of a plain suit.

I now go back to explain how the cards are to be taken up. If it is decided that the cards are to be dealt into 5 and into 6 piles, (the order of the dealing always being immaterial,) you take them up row by row, in consecutive order, from the upper left-hand to the lower right-hand corner. If they are to be dealt into 4, 5 and 6 piles, or into 2 and 7 piles, in any order, you take them up column by column, from the upper right-hand to the lower left-hand corner. The exact reversal of all the cards in the pack will make no difference in the final result. They may also be taken up in columns and dealt into piles whose product is 14 or 39 (as, for example, into 2 piles and 7 piles, or into 3 piles and 13 piles). They may be taken up in rows and dealt into any number of piles whose product is thirty, or, by the multiplication table is ♡4. The following are some of the sets of numbers whose products, counted round a cycle of 53, equal 30: 6·5; 17·8; 7·5·4·4; 9·7·3; 9·8·7·7; 9·6·6·5; 9·9·5·4; X·8·7; X·9·8·7·6; J·J·2; J·8·4·4; J·5·5·3; Q·X·X·4; Q·X·8·5; Q·7·7·6; K·K·3; ♠X·♠X·4 (decimally, 23·13·4); ♠6·♠4·6; ♠5·◊9·◊X.

The products of the following sets count round a cycle of 53 to −30=23; 4·♠6; 2·7·K; K·Q·X; 8·6·6; 9·8·4; X·X·5; Q·J·7; Q·Q·2; 5·5·5·4; 6·4·4·3; X·9·7·5; J·7·6·2; 11·7·4·3; 13·X·6·2; 13·8·5·3; 7·6·5·5·3; 7·7·7·5·4; 9·7·5·5·2; 11·6·5·4·3; 9·8·8·5·4·4; 8·8·7·7·4·4; 11·8·7·7·2·2; 12·11·9·8·7·6.

The products required to prepare the cards for being laid down column by column are ♠6, decimally expressed, 19; and ♡8, decimally expressed, 34.

The following are some of the sets of numbers whose continued products are 19: 9·8; Q·6; 5·5·5; 6·4·3; J·7·3; 13·6·5; 13·10·3; 8·7·6·4; 9·9·8·6; J·9·5·4; 11·10·9·2; 12·8·7·7; 13·10·8·7; 9·8·8·5·4; 10·7·7·6·5; 10·10·10·10·2: 12·7·7·5·5; 7·4·4·4·3; 13·7·4·4·4·4·4·4; 4·4·4·4·4·4·4·4·4·3. The following are sets of numbers whose continued product is 34: ♠4·2; ♠X·K; 29·3; 7·5·4; 9·3·♠♡; 9·9·5: X·7·2·; J·8·4; Q·X·X; 17·11·5; 17·12·9; 19·13·4; 23·11·6: 23·13; 23·17·7; 41·3·2; 5·5·4·4·3; 9·7·7·6·3; 8·6·5·5; 9·9·7·7·2; 13·13·7·2; 17·12·9; 8·4·4·4·4; 2·2·2·2·2·2·2·2·2·2·2·11·10·7·5; 13·12·9; 23·13.

This "trick may be varied in endless ways. For example, you may introduce the derangement that is the inverse of milking. That is, you may pass the cards, one by one, from one hand to the other, placing them alternately att he top and the bottom of the cards held by the

receiving hand. Twelve such operations will bring the cards back to their original order. But a pack of 72 cards would be requisite to show all the curious effects of this mode of derangement.

CHARLES SANTIAGO SANDERS PEIRCE.

MILFORD, PA.