The beta anomaly and mutual fund performance

Paul Irvine
Texas Christian University

Jeong Ho (John) Kim
Emory University

Jue Ren
Texas Christian University

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1 Introduction

The empirical asset pricing literature supplies convincing evidence that high-beta assets often deliver lower expected returns than the CAPM model predicts, and that lower beta assets deliver returns above CAPM expectations (Black, Jensen, and Scholes (1972), Gibbons, Ross, and Shanken (1989), Baker, Bradley, and Wurgler (2011)). Recently, Frazzini and Pedersen (2014) reinvigorate this debate with a compelling theoretical argument for what is broadly termed the beta anomaly. They propose an additional betting against beta (BAB) factor that captures the return spread from this CAPM anomaly.

Given the evidence for the beta anomaly, it has long been suspected that mutual fund managers can capture significant alpha by investing in low-beta stocks. The standard academic response to measuring mutual fund behavior is currently to use the 4-factor model suggested by Carhart (1997). According to this model, in the absence of active management, the expected excess return for a fund is the sum of the products of the betas with four factor risk-premia. The expected difference between the portfolio return and its benchmark return is the Carhart measure of abnormal performance, or the alpha. The Carhart approach in effect assumes that a matching passive portfolio alpha is zero. However, given the current uncertainty regarding the correct multi-factor model to apply to equity returns, and the recent introduction of the BAB factor, whether any asset pricing model effectively controls for the beta anomaly is unclear.

This paper examines whether accounting for the beta anomaly can systematically affect inferences about mutual fund performance. According to the capital asset pricing model, higher mutual fund alpha indicates skill. However, it could also reflect a lower beta exposure to the market. That is, if fund A tends to hold high-beta assets relative to fund B, we ought to expect that, given equal skill, A has a lower alpha than B. In the standard attribution framework, however, we might spuriously attribute this result to differences in skill between A and B. It is not clear a priori how to account for the beta anomaly in mutual fund performance evaluation. More generally, it is not clear how to estimate the value-added of a fund when factor sensitivities are associated with a consistent pattern of alphas. We address the accounting issue by introducing a new performance measure that we call active alpha. Active alpha measure subtracts the passive alpha component from the funds' standard alpha. Passive alpha is measured as the value-weighted alpha of those individual stocks whose betas are similar to estimated fund beta. If the active alpha is positive, investors seeking that particular level of risk would benefit from holding such mutual fund.

In our sample of actively managed U.S. domestic equity funds, we find that alphas are almost monotonically declining in beta for mutual funds, as they are in general for equities. In contrast, we find that active alpha is almost monotonically increasing in beta. It seems apparent that the relation we observe between mutual fund standard alpha and fund beta is a consequence of the beta anomaly. Inference based on our active alpha measure, which accounts for cross-sectional return differences due to the beta anomaly, differs dramatically from that based on standard alpha measures. More specifically, we find that high-beta mutual funds tend to have positive and significant active alpha measures, but low-beta mutual funds tend to have positive and significant standard alpha measures. Moreover, higher active alpha is positively associated with several desirable portfolio characteristics including market-adjusted return and the portfolio Sharpe ratio.

There are other benefits of using active alpha to measure managerial skill. By controlling for passive beta outperformance or underperformance, active alpha controls for any time-variation in average mutual fund beta documented by Boguth and Simutin (2018). Further, adding the BAB factor to excess return models does not appear to completely control for the low-beta anomaly in mutual funds. Although, the magnitude of the alpha-beta relation is smaller in a six-factor model that adds the Pastor and Stambaugh (2003) liquidity factor and the Frazzini and Pedersen (2014) BAB factor to the commonly-used Carhart (1997) four factor model, we find that standard alphas are still significantly negatively related to fund beta.

In our main analysis we show that active alpha is persistent, indicating that it captures some skill over and above allocating assets to low-beta stocks. This finding raises the question of whether investors recognize and respond to active alpha when allocating their cash flows across funds. Related to this question is the fascinating question of what excess return model investors use to allocate their fund flows. Using a Bayesian framework that allows for alternative degrees of belief in different asset pricing models, Busse and Irvine (2006) show that fund flow activity varies by investor beliefs and by the time period under consideration. They report that a 3-year return history has a stronger correlation with fund flows than a single year's performance. Berk and van Binsbergen (2016) use mutual fund flows to test which asset pricing model best fits investor behavior. They test a large number of asset pricing models and time horizons and find that over most, but not all, horizons the CAPM best reflects investor behavior. Barber, Huang, and Odean (2016) find heterogeneous investor responses to fund performance. They report that investors respond most actively to beta risk and treat other factors such as size, value and momentum (the factors in the Carhart (1997) model), as fund alpha. However, they find that more sophisticated investors tend to use more sophisticated benchmarks when evaluating fund performance. Agarwal, Green, and Ren (2017) examine hedge fund flows and find that investors place relatively greater emphasis on exotic risk exposures that can only be obtained from hedge funds. Yet they find little performance persistence in these exotic risks.

Since active alpha is persistent, we investigate how investors allocate their mutual fund flows between standard alpha and active alpha. We find that consistent with the literature, standard alpha generates future fund flows. However, we also find that a subset of investors allocate their fund flows based on our active alpha measure of mutual fund performance. This finding suggests that some mutual fund investors are sophisticated enough to control for the beta anomaly since they invest based on active alpha, a skill measure that controls for portfolio beta. Conversely, we also find investors allocate fund flows based on the passive alpha, or the outperformance that can be obtained by simply generating a low beta portfolio.

To provide an economic explanation for the empirical sensitivity of mutual fund flows to active alpha over and beyond standard alpha, we develop a simple model of mutual fund flows with the presence of both sophisticated and naive investors. In our model, some investors are sophisticated and are able to invest in a passive benchmark with the same risk as the fund. Other investors are naive and only make risky investments via the fund. Both types of investors update the fund's managerial skill as Bayesians. It turns out that sophisticated investors' demand for the fund is positively related to posterior expectations of the active alpha, whereas naive investors' demand for the fund is positively related to posterior expectations of the standard alpha. Intuitively, sophisticated investors consider only active alpha, since they can identify (and short) the passive benchmark portfolio, in turn extracting only the performance truly attributable to managerial ability. On the other hand, naive investors care equally about all sources of alpha, since they are comfortable making risky investments only with the manager.

The model predicts that the flow sensitivities to active alpha and to standard alphas can be either positive or negative, depending on the relative presence of sophisticated investors (and on the persistence of active and passive alphas). Importantly, the empirical fact that flows respond positively to both active alpha and standard alpha measure can be consistent with our rational learning model only given the coexistence of sophisticated and naive investors. The empirical magnitudes of the capital response also suggest that sophisticated investors are relatively rare. We provide supporting evidence for this investor heterogeneity using mutual fund flows from institutional versus retail share classes. As we would expect, it is the flows from institutional share classes that significantly respond to active alpha.

Our paper contributes to the literature on mutual fund performance accounting for return anomalies from the empirical asset pricing literature. Ours is the first to account for the beta anomaly and to produce an estimate of managerial skill that does not attribute skill to a low-beta portfolio tilt. However, the factor-model regression approach is not the only popular mutual fund performance attribution method. The characteristic-based benchmark approach of Daniel, Grinblatt, Titman, and Wermers (DGTW, 1997) is also prominent. Since then, the literature has recognized the importance of accounting for the stock characteristics such as size, value and momentum effects in fund returns. Busse et al. (2017) propose to marry the factor-model regression approach and DGTW approach via a double-adjusted mutual fund performance. Back, Crane, and Crotty (2017) show that fewer funds have significant positive performance than one would expect by chance after alphas are adjusted for coskewness.

As we propose fund beta as a predictor of fund's value, others have proposed fund characteristics that predict performance, including industry concentration in Kacperczyk et al. (2005), the return gap in Kacperczyk et al. (2008), and peer benchmarking in Hunter et al. (2014).

2 Model

We present a model of active management featuring two groups of investors, who will choose to attend to different performance measures when evaluating funds. There is an actively managed mutual fund, whose manager has (potential) ability to generate expected returns in excess of those provided by a passive benchmark—an alternative investment opportunity available to some investors with the same risk as the manager's portfolio. The expected passive alpha on this benchmark and the manager's ability to beat it are unknown to investors, who learn about this ability and the passive alpha by observing the histories of the managed portfolio's returns and the benchmark returns. Let $r_t = \alpha_t + \epsilon_t$ denote the return, in excess of the risk-free rate, on the actively managed fund. This is not the performance attributable to managerial ability, which is α_t net of passive alpha (see below). The parameter α_t is the fund's expected alpha. The error term, ϵ_t, is normally distributed with mean zero and variance σ^2 and is independently distributed through time. We further assume that this uncertainty is systematic: investors cannot diversify away this risk. The passive benchmark portfolio's excess return has mean α_t^P and the same risk as the fund, i.e., $r_t^P = \alpha_t^P + \epsilon_t$.

We note that the model is partial equilibrium. The benchmark portfolio's returns are assumed to be exogenously given, and we do not model the source of successful managers' abilities. In that sense, our approach is similar to that in Berk and Green (2004) and Huang, Wei, and Yan (2012). We are describing the simplest model, which produces the sensitivity of mutual fund flows not only to the standard alpha measure, but also to active alpha that is an alternative measure of fund manager skill controlling for passive alpha.

Specifically, active alpha is the excess return to investors over what would be earned on the passive benchmark, i.e.,

$$
\begin{aligned}
\alpha_t^A &= \alpha_t - \alpha_t^P \\
&= r_t - r_t^P
\end{aligned}
$$

Of course, active alpha is the same as the standard alpha measure if the passive benchmark has zero alpha, but empirical evidence suggests otherwise (e.g., Frazzini and Pedersen, 2014). Note that α_t^A, α_t^P (and in turn α_t) are taken to vary over time. In particular, the active alpha and the passive alpha are assumed to evolve as AR(1)

$$\begin{aligned}
\alpha_t^A &= (1-\gamma)\,\alpha^{A*} + \gamma\alpha_{t-1}^A + \tau_t \\
\alpha_t^P &= (1-\eta)\,\alpha^{P*} + \eta\alpha_{t-1}^P + \upsilon_t,
\end{aligned}$$

where τ_t and υ_t are *i.i.d.*, respectively, $N\left(0,\sigma_\tau^2\right)$ and $N\left(0,\sigma_\upsilon^2\right)$. α^{A*} and α^{P*} represent, respectively, the unconditional expectations of the active alpha and the passive alpha.

There are two types of investors: a fraction q of investors are *sophisticated*, indexed by s, who allocate money across all assets (the risk-free asset and the active fund, as well as its passive benchmark). The remaining $1-q$ fraction of investors are *naive*, indexed by n, who are inexperienced. They only allocate money between the active fund and the risk-free asset. We note that the behavior of naive investors is consistent with the empirical evidence on limited market participation.

On date $t-1$, investors have priors about α_t^A and α_t^P. These investors form their posterior expectations of the fund manager's ability as well as of the passive alpha through Bayesian updating. On date t, after observing the period t excess return r_t, they update their priors about α_t^A and α_t^P, which in turn imply their beliefs about α_{t+1}^A and α_{t+1}^P. Investors' prior beliefs are assumed to be normally distributed with mean $\begin{bmatrix} \widehat{\alpha}_{1|0}^A \\ \widehat{\alpha}_{1|0}^P \end{bmatrix}$ and covariance matrix $\begin{bmatrix} V_{1|0}^A & 0 \\ 0 & V_{1|0}^P \end{bmatrix}$, i.e.,

$$\begin{bmatrix} \alpha_1^A \\ \alpha_1^P \end{bmatrix} \sim N\left(\begin{bmatrix} \widehat{\alpha}_{1|0}^A \\ \widehat{\alpha}_{1|0}^P \end{bmatrix}, \begin{bmatrix} V_{1|0}^A & 0 \\ 0 & V_{1|0}^P \end{bmatrix}\right). \tag{1}$$

Assume that $V_{1|0}^P = \sigma_\upsilon^2$. Then, it is straightforward to show by using standard Bayesian results for updating the moments of a normal distribution that their posterior expectations, $\begin{bmatrix} \widehat{\alpha}^{A}_{t+1|t} \\ \widehat{\alpha}^{P}_{t+1|t} \end{bmatrix}$, after observing the history $\left\{r_u, r^P_u\right\}_{u=1}^{t}$ are:

$$\begin{bmatrix} \alpha^{A}_{t+1} \\ \alpha^{P}_{t+1} \end{bmatrix} \Big| \left\{r_u, r^P_u\right\}_{u=1}^{t} \sim N\left(\begin{bmatrix} \widehat{\alpha}^{A}_{t+1|t} \\ \widehat{\alpha}^{P}_{t+1|t} \end{bmatrix}, \begin{bmatrix} \sigma^2_\tau & 0 \\ 0 & \sigma^2_\upsilon \end{bmatrix} \right)$$

where

$$\widehat{\alpha}^{A}_{t+1|t} = (1-\gamma)\,\alpha^{A*} + \gamma\left(r_t - r^P_t\right) \tag{2}$$

$$\widehat{\alpha}^{P}_{t+1|t} = (1-\eta)\,\alpha^{P*} + \eta w \widehat{\alpha}^{P}_{t|t-1} + \eta\,(1-w)\,r^P_t \tag{3}$$

and $w = \sigma^2_\epsilon / \left(\sigma^2_\upsilon + \sigma^2_\epsilon\right)$. Similarly, this implies the posterior about α_{t+1} is normally distributed with a mean of $\widehat{\alpha}^{A}_{t+1|t} = \left(\widehat{\alpha}^{A}_{t+1|t} + \widehat{\alpha}^{P}_{t+1|t}\right)$ and a variance of $\left(\sigma^2_\tau + \sigma^2_\upsilon\right)$.

We consider an overlapping-generations (OLG) economy in which investors of type $i \in \{s, n\}$ are born each time period t with wealth $W_{i,t}$ and live for two periods. Each time period t, young investors have a constant absolute risk aversion (CARA) utility over their period $t+1$ wealth, $e^{-\gamma_i W_{i,t+1}}$, where $W_{i,t+1} = W_{i,t} + X_{i,t} r_{t+1} + X^P_{i,t} r^P_{t+1}$, $X_{i,t}$ is the dollar allocation to the mutual fund at time t, and $X^P_{i,t}$ is the dollar allocation to the passive benchmark. Naive investors are assumed to make risky investments only with the mutual fund, so that $X^P_{n,t} = 0$.

Given CARA utility, it is easy to show that the optimal mutual fund holdings $X_{s,t}$ and $X_{n,t}$ are

$$X_{s,t} = \frac{\widehat{\alpha}^{A}_{t+1|t}}{\gamma_s \sigma^2_\tau}$$

$$X_{n,t} = \frac{\widehat{\alpha}_{t+1|t}}{\gamma_n \left(\sigma^2_\tau + \sigma^2_\upsilon + \sigma^2_\epsilon\right)}$$

Imposing the restriction that mutual fund holdings cannot be negative (no short selling), we have

$$
\begin{aligned}
X_{s,t} &= \frac{\max\left(\widehat{\alpha}^{A}_{t+1|t}, 0\right)}{\gamma_s \sigma_\tau^2} && (4)\\
X_{n,t} &= \frac{\max\left(\widehat{\alpha}_{t+1|t}, 0\right)}{\gamma_n \left(\sigma_\tau^2 + \sigma_v^2 + \sigma_\epsilon^2\right)} && (5)
\end{aligned}
$$

Intuitively, when choosing their optimal allocation to the fund, sophisticated investors will consider only active alpha, since they have the ability to short the passive benchmark portfolio and in turn extract only the performance truly attributable to managerial ability. On the other hand, naive investors will attend to the standard alpha measure, since they cannot short sell the benchmark asset and in turn care equally about all sources of alpha.

We define the flow into the fund from investors of type i on date t as

$$
\begin{aligned}
F_{i,t} &= \frac{X_{i,t} - X_{i,t-1}\left(1 + r_t\right)}{X_{i,t-1}}. \\
&= \frac{\max\left(\widehat{\alpha}^{A}_{t+1|t}, 0\right) - \max\left(\widehat{\alpha}^{A}_{t|t-1}, 0\right)\left(1 + r_t\right)}{\max\left(\widehat{\alpha}^{A}_{t|t-1}, 0\right)}
\end{aligned}
$$

This

$$
\begin{aligned}
F_{s,t} &= \frac{\max\left(\widehat{\alpha}^{A}_{t+1|t}, 0\right)}{\max\left(\widehat{\alpha}^{A}_{t|t-1}, 0\right)} - \left(1 + r_t\right) \\
F_{n,t} &= \frac{\max\left(\widehat{\alpha}_{t+1|t}, 0\right)}{\max\left(\widehat{\alpha}_{t|t-1}, 0\right)} - \left(1 + r_t\right)
\end{aligned}
$$

The total net flow into the fund is then

$$
\begin{aligned}
F_t &= qF_{s,t} + (1 - q)\, F_{n,t} \\
&= q\frac{\max\left(\widehat{\alpha}^{A}_{t+1|t}, 0\right)}{\max\left(\widehat{\alpha}^{A}_{t|t-1}, 0\right)} + (1 - q)\,\frac{\max\left(\widehat{\alpha}_{t+1|t}, 0\right)}{\max\left(\widehat{\alpha}_{t|t-1}, 0\right)} - \left(1 + r_t\right) && (6)
\end{aligned}
$$

Using (2)-(3), we can derive the following expression for r_t:

$$r_t = \frac{\widehat{\alpha}^A_{t+1|t} - (1-\gamma)\,\alpha^{A*}}{\gamma} + \frac{\widehat{\alpha}^P_{t+1|t} - (1-\eta)\,\alpha^{P*} - \eta w \widehat{\alpha}^P_{t|t-1}}{\eta\,(1-w)}.$$

Plugging this expression into (6), we have

$$F_t = a + q\frac{\max\left(\widehat{\alpha}^A_{t+1|t}\,, 0\right)}{\max\left(\widehat{\alpha}^A_{t|t-1}\,, 0\right)} - \frac{\widehat{\alpha}^A_{t+1|t}}{\gamma} + (1-q)\,\frac{\max\left(\widehat{\alpha}_{t+1|t}\,, 0\right)}{\max\left(\widehat{\alpha}_{t|t-1}\,, 0\right)} - \frac{\widehat{\alpha}^P_{t+1|t}}{\eta\,(1-w)} + \mu_{t-1}$$

where

$$a = \frac{1-\gamma}{\gamma}\alpha^{A*} + \frac{1-\eta}{\eta\,(1-w)}\alpha^{P*} - 1$$

and

$$\mu_{t-1} = \frac{w}{1-w}\widehat{\alpha}^P_{t|t-1}$$

is the time fixed effect.

For easy exposition, we assume the history of observed returns is such that $\widehat{\alpha}^A_{t|t-1}\,, \widehat{\alpha}_{t|t-1} > 0$. Hence, both types of investors started with positive dollar holdings, $X_{s,t-1}, X_{n,t-1} > 0$, in the mutual fund at time $t-1$. Then, the following proposition derives the flow-performance relation that the model delivers.

Proposition 1

$$F_t = a + \beta\widehat{\alpha}_{t+1|t} + \beta^A\widehat{\alpha}^A_{t+1|t} + \mu_{t-1},$$

where

$$\begin{aligned}
\beta &= \frac{(1-q)\,I\left(\widehat{\alpha}_{t+1|t} > 0\right)}{\widehat{\alpha}_{t|t-1}} - \frac{1}{\eta\,(1-w)} \\
\beta^A &= \frac{qI\left(\widehat{\alpha}_{t+1|t} > 0\right)}{\widehat{\alpha}^A_{t|t-1}} - \frac{1}{\gamma}
\end{aligned}$$

and $I(c)$ is an indicator function that equals one if condition c is true and zero otherwise.

Looking forward, it is useful to note two facts that follows immediately from proposition 1. If all investors are naive, i.e., $q = 0$, then the flow sensitivity to active alpha is $-1/\gamma < 0$. On the other hand, if all investors are sophisticated, i.e., $q = 1$, then the flow sensitivity to the standard alpha measure is $-1/\eta(1-w) < 0$. Quintessentially, an empirical observation that flows respond positively to both active alpha and the standard alpha measure would suffice to show that at least some investors are sophisticated and not all investors are sophisticated, i.e., $q \in (0, 1)$. Moreover, how strongly flows respond to active alpha vs. the standard alpha measure would be informative of the fraction of investors who are sophisticated, i.e., how big q is.

3 Data and Methods

3.1 Mutual fund sample

The Morningstar and CRSP merged dataset provides information about mutual fund names, returns, total assets under management (AUM), inception dates, expense ratios, turnover ratios, investment strategies classified into Morningstar Categories, and other fund characteristics. From this data set we collect monthly return and flow data on over 2,838 U.S. diversified equity mutual funds actively managed for the period 1983-2014. Panel A of Table 1 presents summary information about the sample. There are 298,055 fund-month observations. Mean fund size of \$1.28 billion is each fund's total assets under management (AUM), aggregated across share classes, divided by the total stock market capitalization in the same month. To account for the growth over time in the mutual fund industry, we scale this ratio by multiplying it by the total stock market capitalization at the end of 2011 as in Pastor, Stambaugh and Taylor (2015). We compute the fund age from the fund's inception date and find the typical fund has a life of 199 months. Funds earn an average gross return of 0.78% per month and collect fees of 9.9 basis points per month. Monthly firm volatility is 4.64% and average fund beta is 0.99. This beta average suggests that in the fund beta sort results presented below, one can consider the middle decile portfolios to roughly bracket the market beta.

3.1.1 Estimating mutual fund alphas

We estimate the abnormal return (alpha) for each mutual fund using each of the five performance evaluation models: i) the CAPM, ii) the Fama-French (1993) three factor model (FF3), iii) the Carhart (1997) four factor model, iv) the factor model we call PS5 is a five factor model augmenting the Carhart (1997) four-factor model with the Pastor and Stambaugh (2003) liquidity factor as in Boguth and Simutin (2018), and v) the Carhart (1997) four factor model augmented with the Pastor and Stambaugh (2003) liquidity factor and the Frazzini and Pedersen (2014) betting against beta factor (FP6). Alpha estimates are updated monthly based on a rolling estimation window for each model. For example, in the case of the four-factor model for each fund in month t, we estimate the following time-series regression using thirty-six months of returns data from months $\tau = t-1, \ldots t-36$:

$$(R_{p\tau} - R_{f\tau}) = \alpha_{pt} + \beta_{pt}\,(R_{m\tau} - R_{f\tau}) + s_{pt}SMB_\tau + h_{pt}HML_\tau + m_{pt}UMD_\tau + e_{p\tau}, \qquad (7)$$

where $R_{p\tau}$ is the mutual fund return in month τ, $R_{f\tau}$ is the return on the risk-free rate, $R_{m\tau}$ is the return on a value-weighted market index, SMB_τ is the return on a size factor (small minus big stocks), HML_τ is the return on a value factor (high minus low book-to-market stocks), and UMD_τ is the return on a momentum factor (up minus down stocks). The parameters β_{pt}, s_{pt}, h_{pt}, and m_{pt} represent the market, size, value, and momentum tilts (respectively) of fund p; α_{pt} is the mean return unrelated to the factor tilts; and $e_{p\tau}$ is a mean zero error term. (The subscript t

denotes the parameter estimates used in month t, which are estimated over the thirty-six months prior to month t.) We then calculate the alpha for the fund in month t as its realized return less returns related to the fund's market, size, value, and momentum exposures in month t:

$$\widehat{\alpha}_{pt} = (R_{pt} - R_{ft}) - \left[\widehat{\beta}_{pt}(R_{mt} - R_{ft}) + \widehat{s}_{pt}SMB_t + \widehat{h}_{pt}HML_t + \widehat{m}_{pt}UMD_t\right]. \tag{8}$$

We repeat this procedure for all months (t) and all funds (p) to obtain a time series of monthly alphas and factor-related returns for each fund in our sample.

There is an analogous calculation of alphas for other factor models that we evaluate. For example, we estimate a fund's FP6 alpha using the regression of Equation (7), but add the Pastor and Stambaugh (2003) liquidity factor and Frazzini and Pedersen (2014) betting against beta factor as independent variables. To estimate the CAPM alpha, we retain only the market excess return as an independent variable.

3.1.2 Estimating stock alphas

We build the beta-matched passive portfolio from the return characteristics of individual stocks. We estimate abnormal performance for individual stocks in an analogous manner to that of mutual fund alphas described above. First, we estimate the abnormal return (alpha) for each stock using each of the five performance evaluation models. Alpha estimates are updated monthly based on a rolling estimation window. Consider the four-factor model, which includes factors related to market, size, value, and momentum in the estimation of a stock's return. In this case, for each stock in month t, we estimate the following time-series regression using thirty-six months of returns data from months $\tau = t-1, \ldots t-36$where $R_{q\tau}$ is the stock return in month τ, $R_{f\tau}$ is the return on the risk-free rate, $R_{m\tau}$ is the return on a value-weighted market index, SMB_τ is the return on a size factor (small minus big stocks), HML_τ is the return on a value factor (high minus low book-to-market stocks), and UMD_τ is the return on a momentum factor (up minus down stocks). The parameters β_{qt}, s_{qt}, h_{qt}, and m_{qt} represent the market, size, value, and momentum tilts (respectively) of stock q; α_{qt} is the mean return unrelated to the factor tilts; and $e_{q\tau}$ is a mean zero error term. (The subscript t denotes the parameter estimates used in month t, which are estimated over the thirty-six months prior to month t.) We then calculate the alpha for the stock in month t as its realized return less returns related to the stock's market, size, value, and momentum exposures in month t:

$$\widehat{\alpha}_{qt} = (R_{qt} - R_{ft}) - \left[\widehat{\beta}_{qt}(R_{mt} - R_{ft}) + \widehat{s}_{qt}SMB_t + \widehat{h}_{qt}HML_t + \widehat{m}_{qt}UMD_t\right]. \quad (9)$$

We repeat this procedure for all months (t) and all stocks (q) to obtain a time series of monthly alphas and factor-related returns for each stock in our sample.

3.1.3 Estimating mutual fund passive alphas

We calculate the passive alpha for each fund in month t using the alphas and market betas from individual stocks as in Equation (8). The passive alpha for each fund is the value-weighted alpha of those individual stocks whose beta are in a 10 percent range around estimated fund beta, such that:

$$\widehat{\beta}_{qt} > 95\% \times \widehat{\beta}_{pt}, \widehat{\beta}_{qt} < 105\% \times \widehat{\beta}_{pt}. \quad (10)$$

Let $\widehat{\gamma}_{pt}$ denote the esimate of passive alpha for the fund in month t.

3.1.4 Estimating mutual fund active alphas

The fund's passive alpha allows us to calculate the active alpha for the fund in month t as the standard alpha for the fund in month t less the passive alpha in month t :

$$\widehat{\delta}_{pt} = \widehat{\alpha}_{pt} - \widehat{\gamma}_{pt}, \quad (11)$$

where $\widehat{\delta}_{pt}$ is our active alpha estimate for fund p in month t.

3.2 Horizon for performance evaluation

To estimate longer horizon alphas, we cumulate monthly alphas by fund-month. For example, to estimate annual standard alpha:

$$A_{pt} = \prod_{s=0}^{11} (1 + \widehat{\alpha}_{p,t-s}) - 1, \tag{12}$$

where the monthly alpha estimates are calculated from a particular asset pricing model.

Analogously, we calculate the fund's annual active alpha as follows:

$$\Delta_{pt} = \prod_{s=0}^{11} \left(1 + \widehat{\delta}_{p,t-s}\right) - 1, \tag{13}$$

where monthly active alpha estimates can also vary depending on the asset pricing model used as to generate expected returns.

4 Results

4.1 Mutual fund alphas

Table 1 Panel B presents summary information on mutual fund standard alpha, estimated as usual, without correction for any deviation across funds in their betas. Standard alphas are measured against four different asset pricing models that researchers have used to estimate fund performance, the CAPM, the Fama-French 3-factor model (that we designate as FF3), the Carhart 4-factor model (Carhart4), and a five factor model using Carhart's (1997) four factors plus the liquidity factor in Pastor and Stambaugh (2003), that we designate PS5. Average mutual fund standard alphas based on these models are generally less than 2 basis points per month, with the exception of the CAPM, which produces a slightly more positive 6 basis point per month average outperformance. These alphas all represent returns before fees, so that if we subtract the average monthly expense ratio of 9.9 basis points, we see that the average investor underperforms over all the benchmark models.

Panel C of Table 1 present the same statistics for active alpha. Average active alphas are lower than standard alphas for each asset pricing model. After removing the passive alpha component of fund performance, mutual fund managers do not show any degree of stock-picking skill, at least on average. Average active alpha ranges from 1 basis point for the CAPM to -5 basis points for the PS5 benchmark model.

4.2 Mutual fund beta anomaly

Table 2 examines the degree to which mutual fund alphas are exposed to the beta anomaly using four asset pricing models that have been used in the literature to benchmark mutual fund returns. Panel A reports the standard alpha of each fund calculated relative to each asset pricing model. Each month, we sort alphas into 10 portfolios and report the time-series averages by beta decile.

The beta anomaly is clearly evident when examining the standard alphas sorted by beta. Relative to the CAPM, funds in the lowest beta decile have 250 basis points of average outperformance per year, while funds in the highest beta decile underperform by 99 basis points, a large performance spread of 349 basis points. The use of alternative asset pricing models do not lower this spread very much. The often-used Carhart (1997) 4 -factor model reduces the beta 1-10 portfolio spread to 298 basis points. The Fama and French (1993) and the four-factor model augmented with Pastor and Stambaugh (2003) liquidity factor do marginally better than the Carhart (1997) model with 1-10 alpha spreads of 253 basis points and 270 basis points respectively. Mutual fund alphas are all based on gross returns and so do not represent the net-of-fee alpha the investor obtains.

Clearly, if benchmarked with standard alphas, low beta portfolios exhibit a great degree of skill, as evidenced by their outperformance relative to the benchmark models. Panel B reports the results of active alpha, which controls for the beta anomaly affect using a passive beta-matched portfolio (Equation ((11)) to estimate active alpha. The pattern of active alphas is markedly different. Here skill tends to increase with beta, suggesting that high-beta portfolio managers actually exhibit higher skill on average than low-beta portfolio managers once the beta anomaly is controlled for. The active alpha spread is quite large using the CAPM at 331 basis points per year, but the use of the Fama-French 3-factor model, the Carhart (1997) model and the PS5 model do reduce this spread considerably to a minimum of 156 basis points for the PS5 model.

Figure 2 presents the time series of spreads in annualized alpha and active alpha performance between the high-beta and low-beta mutual fund porfolios. All three graphs plot high versus low beta portfolios spreads for standard alpha and active alpha using three different asset pricing models, the CAPM, the Carhart (1997) four-factor model and the FP6 model. As illustrated by the results in Table 2, the active alpha spread, that controls for returns in standard alpha associated with the beta anomaly, is generally positive. Moreover, the active alpha spread is generally larger than the spread in standard alpha.

4.2.1 Multivariate analysis of the beta anomaly and the BAB factor model

Frazzini and Pedersen (2014) contend that the beta anomaly is driven by funding constraints and propose a betting against beta factor (BAB) that captures the return affect related to this particular constraint. Since the BAB factor is intended to control for the low-beta anomaly a relevant question is whether an asset pricing model augmented with the BAB factor removes the performance-beta relation in mutual fund returns. The BAB factor is intended to reflect the tightness of funding constraints driving the beta-anomaly, and to the extent that Frazzini and Pedersen (2014) are correct in explaining the low-beta anomaly, BAB should be related to the size of the anomaly. By extension, BAB should explain at least part of the low-beta premium in mutual fund standard alpha.

We proceed to test the relation between standard alpha and active alpha using both univariate and multivariate regressions in Table 3. Since Table 2 reports that the relation between alpha and beta is similar for all four asset pricing models, for the sake of clarity Table 3 only reports results using the CAPM, the Carhart4, the PS5, and the PS5 model augmented with the BAB factor (FP6) as proxies for expected returns.

The first four columns of Table 3 regress alpha for each of the four asset pricing models on only a constant and beta as a single regressor. The objective in these regressions is to determine the size and significance of the alpha-beta relation documented in Table 2 and to examine whether the addition of the BAB factor to existing asset pricing models controls for this relation in mutual fund returns. Column (1) reports that the coefficient on beta for the CAPM model is -0.05 and statistically significant. This result indicates that if a fund with a beta of 0.5 should expect about an 5% improvement in standard alpha relative to a fund with a beta of 1.5. The results for the Carhart model in column (2) are similar with a slightly larger annual alpha of 6% per year per unit of market risk.

Column (4) reports alpha beta relation using a six-factor model that include the BAB factor (FP6). As expected from Frazzini and Pedersen (2014) the addition of the BAB factor to the benchmark model does reduce the relation between mutual fund alpha and beta. However, the coefficient on beta is 3.1% per year per unit of beta and statistically significant. Despite the use of the BAB factor in the benchmark model, there is still a significant alpha premium to low beta mutual funds. This suggests that including the BAB factor in the existing benchmark models for mutual fund performance does not completely remove the low beta anomaly in mutual fund alphas. Columns (5-8) present multivariate regressions of the same alpha-beta relation, but include controls for fund size and fund age. The coefficients on beta are not significantly affected by the inclusion of these statistically significant controls.

Table 3 of Panel B report the results of identical regressions using active alpha as the dependent variable. In the univariate regressions (Columns 1-4) there is a small positive premium for per unit of beta risk. This could indicate that manager skill is higher in high beta funds, but this relation is only statistically significant using the CAPM. Using the Carhart4, PS5, or FP6 models, the relation is not significant at the 5% level. The multivariate regressions (Columns 5-8) reveal similar results. Only using the CAPM is the relation between beta and active alpha significant, it is insignificant in the Carhart4, the PS5, or the FP6 model regressions.

4.3 Active alpha persistence

Since active alpha is a component of the standard alpha it should be a measure of mutual fund skill distinct from any persistence related to the beta anomaly. If active alpha is a measure of manager skill, it should be repeatable and thus, persistent. We test this contention in Table 4 that presents rank regressions of fund active alpha in month $t-1$, $(\widehat{\delta}_{pt-1})$, against active alpha in the following month, $(\widehat{\delta}_{p,t})$, as well as the next two years $(\widehat{\delta}_{p,t+11}, \widehat{\delta}_{p,t+23})$. These regressions include controls for fund size, expense ratio, fund age, volatility and fund flows to control for fund characteristics that could predict active alpha at time t.

Table 4 Panel A presents regression results using the CAPM model as the base model for calculating active alpha. The results find that active alpha is highly persistent of active alpha in the upcoming month. The rank regression coefficient on $\widehat{\delta}_{p,t-1}$ predicting $\widehat{\delta}_{p,t}$ is 0.897 and statistically significant. This result indicates that a fund earning a high active alpha in month $t-1$

is highly likely to earn a high active alpha in month t. This persistence declines with time as the annual predictability in month $t+11$, is 0.103, yet still statistically significant. Two years out the coefficient on $\widehat{\delta}_{pt-1}$ falls to only 0.006, and is not statistically significant. The control variables in the regression are generally insignificant with the exception of $Volatility$ and fund $Flow$ which show some predictability at the longer horizons, but no control variables are significant at the one-month horizon.

Using the Carhart (1997) four-factor model as the asset pricing model in Panel B produces similar results. Using this model the coefficient of $\widehat{\delta}_{pt-1}$ on $\widehat{\delta}_{pt}$ is 0.898, a number which is slightly higher than the results in Panel A, and indicates significant active alpha predictability at the one month horizon. The level of persistence again declines with time to a statistically significant 0.15 at the one-year horizon and 0.035 at the two-year horizon.

Similar results are obtained when the FP6 model is used as the base model for calculating active alpha in Panel C. Using this model the coefficient of $\widehat{\delta}_{pt-1}$ on $\widehat{\delta}_{pt}$ is 0.900. Statistically significant predictability is also evident at the one- and two- year horizons as well when active alpha is calculated from the FP6 model. Although as in Panels A and B, the coefficients drop dramatically as the horizon increases. Using the FP6 model, the control variables fail to significantly predict active alpha at any horizon.

The results are illustrated graphically in Figure 2. Panel A shows the persitence of active alpha when calculated using the CAPM. Differences in returns persist for about 8 months, though a small amount of outperformance remains until about month 14. Active alpha persistence in the Carhart model is generally smaller, but more persistent as the active alphas in the lowest decile (10) portfolio do not match those in the highest decile (1) portfolio until about month 14. When we use the FP6 asset pricing model, the outperformance of the top active alpha portfolio persists for about 24 months. In summary, regardless of the asset pricing model used to calculate active alpha, the measure shows significant predictability, particularly at shorter horizons.

4.4 Fund performance and active alpha

Table 5 examines the characteristics of active alpha, this persistent skill measure, when benchmarked against the CAPM, the Carhart4, and the FP6 asset pricing models. We do this to better understand how active alpha relates to other mutual fund performance measures.

Panel B of Table 5 presents 10 portfolios sorted by active alpha constructed using the Carhart4 model as the benchmark. Gross returns and market-adjusted returns both increase in the level of active alpha. The 10-1 monthly return spread for both gross and market-adjusted returns is 0.30% per month. The Sharpe ratio also increases as the active alpha increases. Mutual fund monthly Sharpe ratios rise from 0.15 for the lowest active alpha portfolio to 0.21 in the highest active alpha portfolio. The information ratio results mirror the Sharpe ratio results almost exactly. Finally, we find high active alpha portfolios tend to have high standard alpha as well. This is not surprising given we benchmark active alpha against standard alpha (Equation (11)). Active alpha and standard alpha tend to be positively correlated ($\rho = 0.63$). Overall, there is a 0.22% increase in standard alpha as active alpha increases in portfolio rank from low to high.

Panel A of Table 5 presents the results using the CAPM model as the base model for calculating active alpha. Panel C of Table 4 shows the performance of mutual fund portfolios formed based on FP6 active alpha. All of the 10-1 portfolio sort differences are statistically significant and economically meaningful. What this tells us is that a higher active alpha is generally a good thing for portfolio performance. Not only are returns higher as active alpha increases, but portfolio efficiency improves as well.

4.5 Fund flows and active alpha

Table 5 shows that active alpha appears to be a fund characteristic associated with superior portfolio performance, and therefore should be a characteristic cultivated by knowledgeable investors. Table 4 shows that active alpha is persistent, therefore predictable to a large extent by investors. Given these facts, it would make sense for investors to seek out active alpha and allocate their cash flows towards those funds that exhibit high active alpha performance. We also know that some investors allocate their funds based on alpha measures (Barber et al. (2016), Berk and Van Binsbergen (2016)). The natural question then is whether there are any investors that allocate funds based on active alpha.

We begin this investigation in Table 6 that presents the results of panel regressions of fund flow on the lagged ranks of annualized active alpha and standard alpha. We measures fund flows in the standard way as:

$$Flow_t = \frac{TNA_t - TNA_{t-1}(1 + R_t)}{TNA_{t-1}}, \tag{14}$$

so that flows represent the change in net fund assets not attributable to market gains or losses. Table 6 presents the results of a the following regression specification:

$$Flow_t = \alpha + \beta\ Performance_{t-1} + \delta' \mathbf{C}_{t-1} + \epsilon_t, \tag{15}$$

where $Performance_{t-1}$ is measured using either the fund's annualized active alpha ($\widehat{\delta}_{pt-1}$) percentile rank or the fund's annualized standard alpha (α_{t-1}) percentile rank. Control variables ($\mathbf{C}_{t-1}$) include lagged fund flows from month $t-13$ to month $t-1$, lag fund size, the log of fund age, the expense ratio, return volatility and fixed effects for fund style and fund month.

The result based on overall performance rank for Equation (15) are presented in Panel A of

Table 6. We present the coefficients of performance on fund flow in regressions that calculate the standard alpha and active alpha from three different asset pricing models, the CAPM, the Carhart four-factor model and the six-factor model we designate FP6 that contains the BAB factor. Since active alpha is, by our construction, a component of standard alpha, we estimate the effect of alpha and active alpha independently to better understand the strength of each measure in capturing fund flows.[1] Fund flows are significantly positively related to past performance measured either by alpha or by active alpha. For both performance measures, the flow-performance relation weakens slightly as more factors are added to the asset pricing model, but in all six regressions alpha and active alpha significantly attract fund flows. The coefficients on active alpha are approximately two-thirds of the size of those on standard alphas. This result indicates that the skill component of standard alpha attracts significant inflows, but also that there is a significant fraction of investors that allocate flows to the passive component of standard alpha. This relative strength of the coefficients is not surprising as standard alpha is a more familiar performance measure, but the results do indicate that there exists a subset of investors who, are apparently aware that passive alpha should not necessarily be rewarded, and allocate flows based on active alpha.

Panel B is identical to Panel A except that in this Panel we follow the practice of Sirri and Tufano (1998), who examine percentile ranked performance measure within investment style categories. To estimate the effect of performance within style categories, we first calculate the percentile rank within each style. The results are similar for this alternative ranking of active alpha and standard alpha performance. Flows are allocated to both alpha and active alpha. The strength of the coefficient on active alpha is positive and significant yet smaller than the coefficient on standard alpha. Within investment styles, investors are allocating significant fund flows to the fund's active alpha, but investors are more strongly attracted to fund's producing high standard alphas, an

[1] Below in Table 7, we present estimate the relative strenght of the two components of standard alphas in attracting fund flows.

indication that some investors are allocating funds based on the passive component of standard alpha. This result holds even when using the FP6 asset pricing model, a model that includes the betting againt beta factor of Frazzini and Pedersen (2014).

When we look at the results in Table 6 it is apparent that any flows chasing the standard alpha measure, regardless of the assumed return generating process, are attracted by either the passive alpha component of standard alpha or the active alpha component. Therefore any flows allocated to alpha are either allocated to the passive alpha obtained from the beta anomaly or the active alpha component. We maintain that any flows allocated to passive alpha are not rewarding managerial outperformance, instead they are rewarding the inability of the asset pricing model to fully control for the beta anomaly. Table 7 estimates how fund flows are associated with the two components of standard alpha, the active alpha and the passive alpha, as well as the six factors in the FP6 asset pricing model.

The results presented in Table 7 illustrate how all returns, whatever the source, tend to attract fund flows. These results suggest that mutual fund investors allocate some of their flows based on potential asset pricing factors as in Barber et al. (2016),but our results indicate that both the liquidity and betting against beta factor also attract fund flows, factors that were not examined in Barber et al. (2016) Active alpha and passive alpha both attract statistically significant flows into the fund at approximately the same rate when included as regressors in the same specification for fund flows. The coefficient on active alpha is 0.151 and the coefficient on passive alpha is 0.150. This result indicates that after controlling for factor returns, some investors are allocating flows based on the passive alpha component of standard alphas, a measure that we argue should not be attributed to managerial skill. Looking at the performance of specific factors in attracting fund flows, the market return has the weakest coefficient on fund flows, while the more exotic returns associated with the momentum and liquidity factors attract funds at the greatest rate.

4.6 Who invests in active alpha?

Thus far, we have provided evidence that some investors allocate flows to managers who exhibit high active alpha performance. In this section, we test and find support for the conjecture that sophisticated investors tend to use active alpha measure. As in Evans and Fahlenbrach (2012), we use institutional share class as a proxy for investor sophistication .

We test the impact of institutional share class on the flow-return relations. To do so, we first classify a mutual fund share class as institutional if Morningstar share class is INST or CRSP institution dummy is 1. For each mutual fund, we measure the flow to its institutional class as the value-weighted flow across fund's multiple institutional classes. Similar, the flow to fund's retail share class is the value-weighted flow across fund's retail classes. Finally, we modify the main flow-return regression Equation (15) by including an interaction term between active alpha and the institution share class dummy.

Table 8 presents regression coefficient estimates from panel regressions of monthly flow to institution/retail share class (dependent variable) on lagged rank of annualized active alpha and the interaction term with institution share class dummy variable. Panel A reports the regression results for all of the mutual funds in our sample over the period 1984 to 2014. The interaction term between active alpha and institutional share class dummy is significant at 1% level. In Panel B, we restrict the sample to mutual funds with both institutional and retail share classes. We consistently find that investors in the institutional share classes respond more to active alpha than do investors in the retail share classes. These results are consistent with the notion that investors in the retail share classes are less sophisticated in their assessment of funds performance than are investors in the institutional share class. Overall, these results confirm our hypothesis that sophisticated investors allocate flows to mutual funds that exhibit high active alpha performance.

5 Conclusion

Mutual fund managers can earn positive alphas passively by allocating resources to low beta assets to take advantage of the low-beta anomaly. This positive relation between beta and standard alpha is significant over a number of different asset pricing models, including a six-factor model that includes the four factors in the Carhart (1997) model plus a liquidity factor and the betting against beta factor of Frazzini and Pedersen (2014). To correct for the passive alphas that can be recorded regardless of the asset pricing model, we develop a measure of alpha called active alpha that subtracts the outperformance from a beta-matched portfolio from the fund's standard alpha. We contend that active alpha is a useful measure of managerial skill since it isolates outperformance that is distinct from the outperformance that can be obtained from the low-beta anomaly.

A high active alpha is associated with positive portfolio properties including overall returns, market-adjusted returns and high Sharpe ratios. Active alpha is also predictable, in that past active alphas are significantly correlated with future active alphas for at least 12 months into the future. Given the positive properties of high active alpha portfolios and the fact that it is to some extent predictable, sophisticated investors should allocate their capital to high active alpha funds. We find evidence that active alpha does attract cash flows, particularly from more sophisticated investors who are presumably aware of the low-beta anomaly.

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Table 1: Summary Statistics

This table summarizes the statistics across fund-month observations from Jan. 1983 to Dec. 2014. Panel A reports fund characteristics such as net return, flows, fund size, expense ratio, age, and volatility. Percentage fund flow is percentage change TNA from month t-1 to t adjusted for the fund return in month t. Volatility is calculated as the standard deviation of prior 12 month fund returns. All variables are winsorized at the 1% and 99% levels. Panel B presents the estimated alphas from 36-month rolling regressions using various factor models. Panel C presents of estimated active alphas using various factor models.

	#obs	Mean	SD	25th perc	Median	75th perc
Panel A: Fund Characteristics						
Monthly net return	298,055	0.779%	5.130%	-1.870%	1.250%	3.830%
Percentage fund flow	298,055	-0.097%	4.160%	-1.490%	-0.409%	0.832%
Fund size ($mil)	298,055	1,277	2,838	105.5	324.6	1,041
Expense ratio (per month)	298,055	0.098%	0.110%	0.078%	0.096%	0.117%
Age (months)	298,055	198.9	161.7	92	147	238
Volatility (t-12 to t-1)	298,055	4.635	2.093	3.03	4.231	5.769
Fund Beta	298,055	0.998	0.165	0.909	0.998	1.083
Panel B: Fund Performance - Standard Alpha (per month)						
CAPM alpha	298,055	0.059%	2.310%	-0.997%	0.019%	1.050%
FF3 alpha	298,055	0.005%	1.830%	-0.866%	-0.001%	0.860%
Carhart4 alpha	298,055	-0.006%	1.800%	-0.856%	-0.007%	0.840%
PS5 alpha	298,055	0.003%	1.830%	-0.854%	0.004%	0.859%
Panel C: Fund Performance - Active Alpha (per month)						
CAPM active alpha	297,926	0.011%	3.020%	-1.530%	-0.019%	1.500%
FF3 active alpha	297,977	-0.048%	2.620%	-1.450%	-0.046%	1.370%
Carhart4 active alpha	297,981	-0.038%	2.640%	-1.380%	-0.032%	1.350%
PS5 active alpha	297,970	-0.050%	2.620%	-1.410%	-0.026%	1.340%

Table 2: Beta Anomaly in Mutual Fund Returns

Panel A reports average annualized abnormal return (alpha) for deciles of mutual funds sorted according to market beta exposures. We estimate the beta for a mutual fund using each of the four performance evaluation models. Market beta exposures are updated monthly based on a rolling regression using prior thirty-six months of returns data. Panel B reports average annualized active alphas for deciles of mutual funds sorted according to market beta exposures. Each month, alphas and active alphas are estimated according to Section xx for each of the four performance evaluation models. We use Newey-West (1987) standard errors with 18 lags; t-statistics are presented in parentheses. *, **, and *** denote 10%, 5%, and 1% significance, respectively.

	Panel A: Alpha				Panel B: Active Alpha			
Beta Group	CAPM	FF3	Carhart4	PS5	CAPM	FF3	Carhart4	PS5
1 (low)	2.50%	1.80%	1.80%	1.81%	-1.02%	-1.29%	-1.42%	-1.27%
2	1.56%	1.10%	0.92%	0.94%	-1.52%	-1.01%	-0.74%	-0.49%
3	1.06%	0.91%	0.55%	0.77%	-1.40%	-1.46%	-0.18%	-1.37%
4	0.72%	0.70%	0.46%	0.64%	-0.34%	-1.04%	0.17%	-1.10%
5	0.52%	0.54%	0.35%	0.48%	-0.69%	-0.50%	-0.26%	-0.43%
6	0.60%	0.19%	0.03%	0.19%	0.35%	-0.13%	-0.83%	-0.45%
7	0.50%	0.08%	-0.16%	0.03%	0.97%	0.30%	-0.82%	-0.12%
8	-0.04%	-0.18%	-0.44%	-0.19%	0.57%	0.07%	-0.74%	0.04%
9	-0.36%	0.19%	-0.19%	-0.17%	1.36%	0.67%	0.06%	0.42%
10 (high)	-0.99%	-0.73%	-1.18%	-0.89%	2.29%	1.19%	0.59%	0.30%
High-Low	-3.48%	-2.53%	-2.98%	-2.69%	3.31%	2.48%	2.01%	1.56%

Table 3: Beta Anomaly in Mutual Fund Returns

Panel A shows the Fama-MacBeth (1973) estimates of annualized alpha regressed on market beta exposure. We estimate the beta for a mutual fund using each of the four performance evaluation models. Panel B shows the Fama-MacBeth (1973) estimates of annualized active alpha regressed on market beta exposure. Market beta exposures are updated monthly based on a rolling regression using prior thirty-six months of returns data. The controls in Column (1) - (4) are nine Morningstar category dummies. The controls in Column (5) - (8) are nine Morningstar category dummies, size, and age. Standard errors are presented in parentheses. *, **, and *** denote 10%, 5%, and 1% significance, respectively.

Panel A: Alpha	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	CAPM	Carhart4	PS5	FP6	CAPM	Carhart4	PS5	FP6
Beta	-0.050**	-0.060***	-0.058***	-0.031**	-0.051**	-0.061***	-0.059***	-0.032***
	(0.020)	(0.012)	(0.012)	(0.013)	(0.020)	(0.012)	(0.012)	(0.012)
Size					0.002***	0.002***	0.002***	0.002***
					(0.000)	(0.001)	(0.001)	(0.000)
Age					-0.003**	-0.004***	-0.004***	-0.004***
					(0.001)	(0.000)	(0.001)	(0.001)
Constant	0.063***	0.067***	0.066***	0.0422***	0.062***	0.072***	0.073***	0.053***
	(0.023)	(0.011)	(0.011)	(0.012)	(0.022)	(0.010)	(0.010)	(0.011)
Style fixed effects	YES	YES	YES	YES	YES	YES	YES	YES
Observations	270,266	270,266	270,266	270,266	269,714	269,714	269,714	269,714
R-squared	0.360	0.148	0.143	0.122	0.371	0.160	0.156	0.134

Panel B: Active Alpha	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	CAPM	Carhart4	PS5	FP6	CAPM	Carhart4	PS5	FP6
Beta	0.036***	0.032*	0.029	0.021	0.035***	0.030	0.027	0.020
	(0.013)	(0.019)	(0.020)	(0.019)	(0.013)	(0.019)	(0.019)	(0.019)
Size					0.002***	0.002***	0.002***	0.002***
					(0.000)	(0.000)	(0.001)	(0.001)
Age					-0.004***	-0.004***	-0.004***	-0.004***
					(0.001)	(0.001)	(0.001)	(0.001)
Constant	-0.030**	-0.029	-0.027	-0.019	-0.025	-0.024	-0.019	-0.007
	(0.014)	(0.019)	(0.021)	(0.020)	(0.016)	(0.018)	(0.019)	(0.020)
Style fixed effects	YES	YES	YES	YES	YES	YES	YES	YES
Observations	270,119	270,179	270,159	270,153	269,567	269,627	269,607	269,601
R-squared	0.222	0.110	0.104	0.099	0.232	0.122	0.114	0.110

Table 4: Active Alpha Persistence

This table reports the results of Fama-MacBeth regressions of the future annualized active alpha rank on the past annualized active alpha rank. A fund's rank represents its percentile performance relative to other funds within the same Morningstar-style box during each month. The rank ranges from 0 to 1. Column (2) reports the monthly cross-sectional regression of annualized active alpha rank on prior month's annualized active alpha rank. We use Newey-West (1987) standard errors with twenty-four lags for column (2). Column (3) reports the monthly cross-sectional regression of annualized active alpha rank on prior year's annualized active alpha rank. We use Newey-West (1987) standard errors with eighteen lags for column (3). Column (4) reports the monthly cross-sectional regression of annualized active alpha on prior two year's annualized active alpha rank. We use Newey-West (1987) standard errors with twelve lags for column (4). In Panel A, active alpha is based on CAPM model. In Panel B, active alpha is based on four-factor model. In Panel C, active alpha is based on six-factor model. Standard errors are presented in parentheses. *, **, and *** denote 10%, 5%, and 1% significance, respectively.

Panel A: CAPM Active Alpha

	Active Alpha$_t$	Active Alpha$_{t+11}$	Active Alpha$_{t+23}$
Active Alpha$_{t-1}$	0.897***	0.103***	0.006
	(0.004)	(0.013)	(0.014)
Log Fund size	0.000	0.000	0.002
	(0.000)	(0.003)	(0.003)
Log Exp. Ratio	-0.079	-0.118	0.342
	(0.116)	(0.785)	(0.816)
Log Age	-0.001	-0.004	-0.001
	(0.001)	(0.003)	(0.004)
Volatility	0.0017	0.012**	0.011**
	(0.001)	(0.005)	(0.005)
Flow	-0.013	-0.342***	-0.094**
	(0.024)	(0.132)	(0.043)
Constant	0.049***	0.424***	0.443***
	(0.006)	(0.037)	(0.036)
R-squared	0.817	0.056	0.039
Observations	267,316	245,578	221,225
Correlation	0.901	0.103	0.018

Table 4 continued

Panel B: Carhart4 Active Alpha

	Active Alpha$_t$	Active Alpha$_{t+11}$	Active Alpha$_{t+23}$
Active Alpha$_{t-1}$	0.898***	0.150***	0.0347***
	(0.003)	(0.014)	(0.010)
Log Fund size	0.000	0.0013	0.002
	(0.000)	(0.003)	(0.003)
Log Exp. Ratio	-0.154	-1.154	-1.233
	(0.100)	(0.729)	(0.819)
Log Age	-0.001	-0.009**	-0.011**
	(0.001)	(0.004)	(0.004)
Volatility	0.001	0.007	0.004
	(0.001)	(0.005)	(0.005)
Flow	0.005	-0.168**	-0.099
	(0.02)	(0.084)	(0.060)
Constant	0.052***	0.453***	0.540***
	(0.005)	(0.028)	(0.038)
R-squared	0.817	0.068	0.043
Observations	267,374	245,616	221,260
Correlation	0.901	0.146	0.040

Panel C: FP6 Active Alpha

	Active Alpha$_t$	Active Alpha$_{t+11}$	Active Alpha$_{t+23}$
Active Alpha	0.900***	0.152***	0.059***
	(0.004)	(0.014)	(0.011)
Log Fund size	0.000	-0.001	-0.001
	(0.000)	(0.002)	(0.003)
Log Exp. Ratio	-0.059	-0.359	-0.763
	(0.114)	(0.830)	(0.720)
Log Age	-0.001	-0.006	-0.006
	(0.001)	(0.004)	(0.005)
Volatility	0.000	0.0054	0.005
	(0.001)	(0.004)	(0.004)
Flow	0.017	-0.144*	-0.019
	(0.015)	(0.079)	(0.068)
Constant	0.054***	0.447***	0.495***
	(0.005)	(0.027)	(0.035)
R-squared	0.820	0.064	0.042
Observations	267,349	245,616	221,260
Correlation	0.900	0.130	0.047

Table 5: Active Alpha Sort Portfolio

This table reports performance of active-alpha sorted calendar-time mutual fund portfolios. Each month, mutual funds are assigned to one of ten deciles mutual fund portfolios based on prior month's annualized active alpha. Panel A reports CAPM active alpha sort results. Panel B reports Carhart4 active alpha sort results. Panel C reports FP6 active alpha sort results. All mutual funds are equally weighted within a given portfolio, and the portfolios are rebalanced every month to maintain equal weights. column (2) - column (5) report mutual fund portfolio's time series average of gross return, market adjusted return, Sharpe ratio, information ratio, and Carhart4 alpha. We use Newey-West (1987) standard errors with eighteen lags; t-statistics are presented in parentheses. *, **, and *** denote 10%, 5%, and 1% significance, respectively.

Panel A: CAPM Active Alpha

Active Alpha	Gross Ret	MAR	Shar. R.	Info. R.	Carhart4 Alpha
1 (low)	0.901%	-0.175%	0.1484	0.1479	-0.045%
2	0.949%	-0.122%	0.1653	0.1649	-0.030%
3	0.958%	-0.111%	0.1683	0.1679	-0.018%
4	0.959%	-0.110%	0.1669	0.1666	-0.025%
5	0.973%	-0.096%	0.1689	0.1685	-0.018%
6	0.988%	-0.081%	0.1740	0.1737	0.010%
7	1.026%	-0.044%	0.1859	0.1857	0.021%
8	1.064%	-0.006%	0.1914	0.1911	0.037%
9	1.110%	0.038%	0.2036	0.2034	0.068%
10 (high)	1.199%	0.120%	0.2042	0.2039	0.116%
High-Low	0.298%***	0.295%***	0.0557**	0.0560**	0.161%
t-stats	(2.636)	(2.602)	(2.057)	(2.064)	(0.949)

Panel B: Carhart4 Active Alpha

Active Alpha	Gross Ret	MAR	Shar. R.	Info. R.	Carhart4 Alpha
1 (low)	0.906%	-0.173%	0.1483	0.1479	-0.058%
2	0.975%	-0.097%	0.1643	0.1638	-0.006%
3	0.991%	-0.079%	0.1722	0.1719	-0.017%
4	0.983%	-0.086%	0.1731	0.1728	-0.015%
5	0.995%	-0.073%	0.1738	0.1735	0.006%
6	0.999%	-0.070%	0.1748	0.1744	0.009%
7	1.002%	-0.066%	0.1820	0.1817	-0.002%
8	1.026%	-0.044%	0.1859	0.1856	0.023%
9	1.044%	-0.027%	0.1889	0.1886	0.012%
10 (high)	1.206%	0.128%	0.2112	0.2110	0.166%
High-Low	0.300%***	0.301%***	0.0629***	0.0631***	0.224%***
t-stats	(3.000)	(3.022)	(2.906)	(2.911)	(2.673)

Table 5 continued

Panel C: FP6 Active Alpha

Active Alpha	Gross Ret	MAR	Sharpe R	Info. R	Carhart4 Alpha
1 (low)	0.976%	-0.101%	0.1639	0.1635	0.001%
2	0.990%	-0.080%	0.1697	0.1693	0.002%
3	0.986%	-0.083%	0.1758	0.1754	0.000%
4	0.984%	-0.085%	0.1685	0.1681	-0.037%
5	0.961%	-0.107%	0.1704	0.1701	-0.015%
6	0.996%	-0.073%	0.1770	0.1767	-0.011%
7	0.983%	-0.086%	0.1731	0.1728	-0.022%
8	1.005%	-0.065%	0.1769	0.1766	-0.001%
9	1.061%	-0.012%	0.1941	0.1939	0.063%
10 (high)	1.182%	0.102%	0.2056	0.2053	0.134%
High-Low	0.205%***	0.203%***	0.0417***	0.0419***	0.133%**
t-stats	(2.658)	(2.639)	(2.461)	(2.459)	(1.967)

Table 6: Fund Flows and Competing Measures

This table presents regression coefficient estimates from panel regressions of monthly fund flow (dependent variable) on the lagged rank of annualized active alpha and the lagged rank of annualized alpha. In Panel A, a mutual fund's rank represents its percentile performance relative to all other mutual funds during the same month. In Panel B, a fund's rank represents its percentile performance relative to other funds within the same Morningstar-style box during the same month. The rank ranges from 0 to 1. We calculate the rank based on annualized alpha and annualized active alphas. Controls include lagged fund flows from month t-13, lagged values of log of fund size, log of fund age, expense ratio, return volatility, and style-month fixed effects. Standard errors (double-clustered by fund and month) are presented in parentheses. *, **, and *** denote 10%, 5%, and 1% significance, respectively.

Dependent Variable: Flow

Panel A (1): Overall Alpha Percentile Rank				Panel A (2): Overall Active Alpha Percentile Rank			
	CAPM	Carhart4	FP6		CAPM	Carhart4	FP6
Alpha	0.039***	0.029***	0.027***	Active Alpha	0.025***	0.020***	0.019***
	(0.001)	(0.001)	(0.001)		(0.001)	(0.001)	(0.001)
Observations	269,610	269,610	269,610		269,463	269,523	269,497
R-squared	0.243	0.232	0.225		0.219	0.213	0.209
Style-month fix effects	YES	YES	YES		YES	YES	YES
Controls	YES	YES	YES		YES	YES	YES

Panel B (1): Alpha Percentile Rank within Style				Panel B(2): Active Alpha Percentile Rank within Style			
	CAPM	Carhart4	FP6		CAPM	Carhart4	FP6
Alpha	0.031***	0.027***	0.025***	Active Alpha	0.022***	0.019***	0.018***
	(0.001)	(0.001)	(0.001)		(0.001)	(0.001)	(0.001)
Observations	269,610	269,610	269,610		269,463	269,523	269,497
R-squared	0.243	0.231	0.225		0.219	0.212	0.209
Style-month fix effects	YES	YES	YES		YES	YES	YES
Controls	YES	YES	YES		YES	YES	YES

Table 7: Fund Flow Response to Fund Return Components

This table presents regression coefficient estimates from panel regressions of monthly fund flow (dependent variable) on the components of a fund's return - a funds' active alpha, passive alpha, and six factor-related return. Factor-related returns are estimated based on the funds' factor exposure and the factor return. The six factors include the market, size, value, momentum, liquidity, and betting-against-beta. Controls include lagged fund flows from month t-13, lagged values of log of fund size, log of fund age, expense ratio, return volatility, and style-month fixed effects. Standard errors (double-clustered by fund and month) are presented in parentheses. *, **, and *** denote 10%, 5%, and 1% significance, respectively.

Dependent Variable: Flow	
Active Alpha	0.151***
	(0.006)
Passive Alpha	0.150***
	(0.007)
Mkt Ret	0.051***
	(0.008)
Size Ret	0.117***
	(0.013)
Value Ret	0.092***
	(0.010)
Mom Ret	0.121***
	(0.008)
Liquidity Ret	0.196***
	(0.014)
Bab Ret	0.112***
	(0.011)
Observations	269,497
R-squared	0.246
Style-month fixed effects	YES
Controls	YES

Table 8: Investor Sophistication and Flow-Active Alpha Relationship

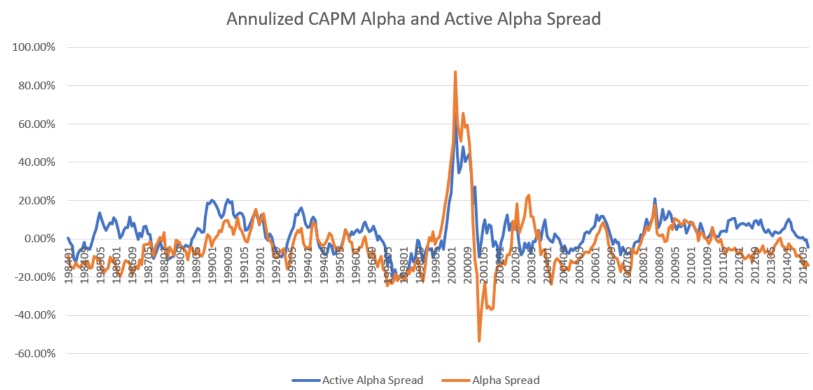
This table presents regression coefficient estimates from panel regressions of monthly flow to institution/retail share class (dependent variable) on lagged rank of annualized active alpha and the interactions with institution share class dummy variable. A share class is defined as institutional share class if Morningstar share class is INST or CRSP institution fund dummy is 1. For each mutual fund, the flow to its institutional class is the value-weighted flow across fund's multiple institutional classes. Similar, the flow to fund's retail share class is the value-weighted flow across fund's retail classes. Panel A the regression result for all of the mutual funds in our sample over the period 1984 to 2014. In Panel B, we restrict the sample to mutual funds with both institutional and retail share classes. Controls include lagged rank of annualized alpha, lagged fund flows from month t-13, lagged values of log of fund size, log of fund age, expense ratio, return volatility, and style-month fixed effects. Standard errors (double-clustered by fund and month) are presented in parentheses. *, **, and *** denote 10%, 5%, and 1% significance, respectively.

Dependent Variable: Flow

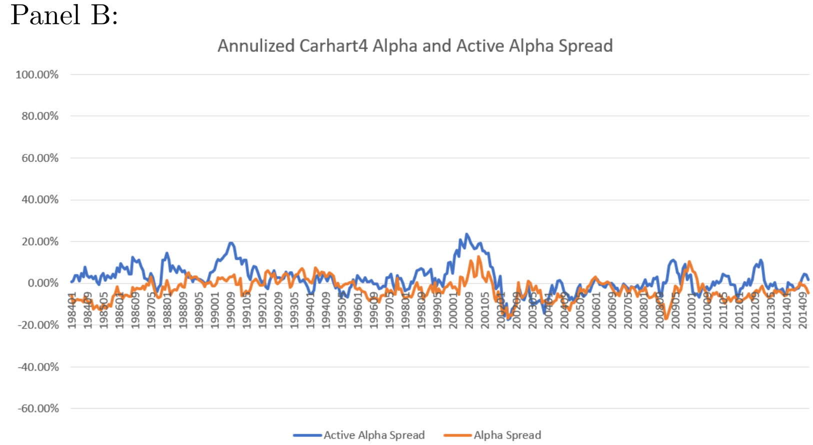
Panel A: Full Sample				Panel B: Smaller Sample		
	CAPM	Carhart4	FP6	CAPM	Carhart4	FP6
Active Alpha	0.020***	0.016***	0.014***	0.019***	0.014***	0.012***
	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	(0.002)
Inst. Class * Active Alpha	0.024***	0.022***	0.023***	0.025***	0.023***	0.024***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Log Fund size	0.001***	0.001***	0.001***	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Log Exp. Ratio	0.450***	0.507***	0.468***	0.313*	0.389**	0.348**
	(0.090)	(0.091)	(0.090)	(0.162)	(0.163)	(0.162)
Log Age	-0.002***	-0.002***	-0.002***	-0.001	-0.001	-0.001
	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)
Volatility	-0.002***	-0.002***	-0.002***	-0.003***	-0.002***	-0.002***
	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)
Lagged Flow	0.155***	0.152***	0.152***	0.185***	0.182***	0.182***
	(0.007)	(0.007)	(0.007)	(0.010)	(0.011)	(0.011)
Constant	0.008	-0.023	0.100***	0.012	-0.000	-0.167
	(0.031)	(0.035)	(0.003)	(0.021)	(0.000)	(0.222)
R-squared	0.058	0.055	0.055	0.040	0.037	0.037
Observations	382,780	382,867	382,815	226,635	226,689	226,637
Style-month fixed effects	YES	YES	YES	YES	YES	YES

Figure 1: Time series of spreads between high beta and low beta mutual fund portfolios
This figure plots the annualized alpha and annualized active alpha spreads between highest beta portfolio and lowest beta decile portfolio (decile 10 - decile1). Each month, mutual funds are ranked into equal-weight decile portfolios based on market beta exposures estimate. Mutual fund alphas, active alphas, and market beta exposures are updated monthly based on a rolling regression using prior thirty-six months of returns data. In Panel A, we report annualized alpha and active alpha spreads based on CAPM; in the middle Panel B, we report annualized alpha and active alpha spreads based on Carhart four-factor model; and in Panel C, we report annualized alpha and active alpha spreads based on FP six-factor model.

Panel A:



Panel B:



Panel C:

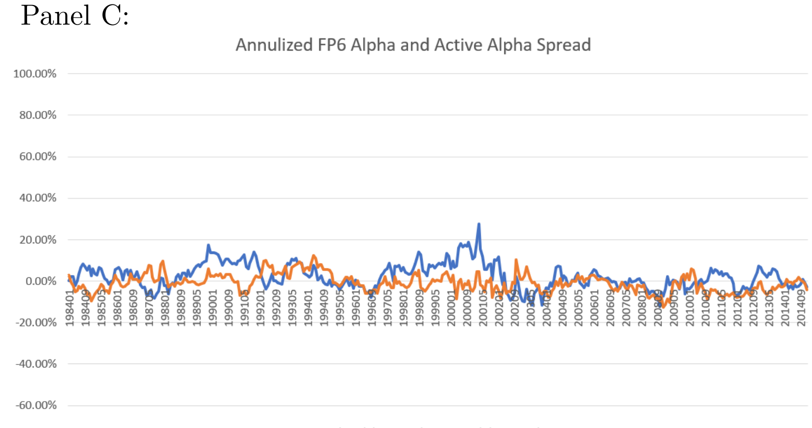
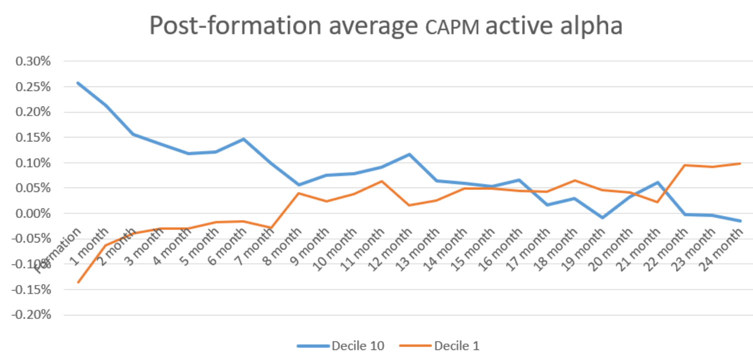


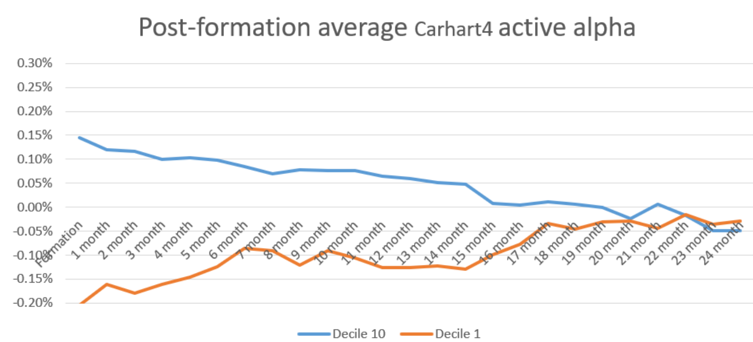
Figure 2: Persistence of Active Alpha

This figure depicts the average monthly active alpha of portfolios tracked over a 2-year period between 1984 and 2014. The portfolios are formed by sorting all the funds into deciles according to lagged annualized active alpha. Subsequently, the top and bottom decile portfolios are tracked over the next 2-year period. The portfolios are equally weighted each month, so the are readjusted whenever a fund disappears from the sample. In Panel A, we report the CAPM active alphas; in Panel B, we report the Carhart4 active alpha; and in Panel C, we report the FP6 active alpha.

Panel A:



Panel B:



Panel C: