

SYNTHESIS OF LITERATURE, LOSS-RATE FUNCTIONS, REGIONAL CHARACTERISTICS OF UNIT HYDROGRAPHS

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Regional Characteristics of Unit Hydrographs**

by

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APPROXIMATE CONVERSIONS TO SI UNITS

APPROXIMATE CONVERSIONS FROM SI UNITS

Symbol	When You Know	Multiply By	To Find	Symbol	When You Know	Multiply By	To Find	Symbol
LENGTH								
in	inches	25.4	millimeters	mm	millimeters	0.039	inches	in
ft	feet	0.305	meters	m	meters	3.28	feet	ft
yd	yards	0.914	meters	m	meters	1.09	yards	yd
mi	miles	1.61	kilometers	km	kilometers	0.621	miles	mi
AREA								
in ²	square inches	645.2	square millimeters	mm ²	square millimeters	0.0016	square inches	in ²
ft ²	square feet	0.093	square meters	m ²	square meters	10.764	square feet	ft ²
yd ²	square yards	0.836	square meters	m ²	square meters	1.195	square yards	yd ²
ac	acres	0.405	hectares	ha	hectares	2.47	acres	ac
mi ²	square miles	2.59	square kilometers	km ²	square kilometers	0.386	square miles	mi ²
VOLUME								
fl oz	fluid ounces	29.57	milliliters	mL	milliliters	0.034	fluid ounces	fl oz
gal	gallons	3.785	liters	L	liters	0.264	gallons	gal
ft ³	cubic feet	0.028	cubic meters	m ³	cubic meters	35.71	cubic feet	ft ³
yd ³	cubic yards	0.765	cubic meters	m ³	cubic meters	1.307	cubic yards	yd ³
NOTE: Volumes greater than 1000 l shall be shown in m ³ .								
MASS								
oz	ounces	28.35	grams	g	grams	0.035	ounces	oz
lb	pounds	0.454	kilograms	kg	kilograms	2.202	pounds	lb
T	short tons (2000 lb)	0.907	megagrams (or "metric ton")	Mg (or "t")	megagrams (or "metric ton")	1.103	short tons (2000 lb)	T
TEMPERATURE (exact)								
°F	Fahrenheit temperature	5(F-32)/9 or (F-32)/1.8	Celcius temperature	°C	Celcius temperature	1.8C + 32	Fahrenheit temperature	°F
ILLUMINATION								
fc	foot-candles	10.76	lux	lx	lux	0.0929	foot-candles	fc
fl	foot-Lamberts	3.426	candela/m ²	cd/m ²	candela/m ²	0.2919	foot-Lamberts	fl
FORCE and PRESSURE or STRESS								
lbf	poundforce	4.45	newtons	N	newtons	0.225	poundforce	lbf
lbf/in ²	poundforce per square inch	6.89	kilopascals	kPa	kilopascals	0.145	poundforce per square inch	lbf/in ²

* SI is the symbol for the International System of Units. Appropriate

(Revised September 1993)

**Synthesis of Literature
Regional Characteristics of Unit Hydrographs
TxDOT Project 0-4193**

Introduction

A hydrograph is a time-series of either water-surface elevation or instantaneous discharge, taken at a particular point on a stream. The point on the stream represents the outlet from a watershed, or the area topographically above that point from which all drainage passes through that point. The hydrograph represents the integrated response of the watershed to precipitation and all other hydrometeorologic processes extant on the watershed. Hydrologists and engineers use hydrographs to analyze the characteristics of a watershed and for design of engineering projects.

In particular, when watershed drainage area exceeds 200 acres, but is less than about 20 square miles, TxDOT engineers use the hydrograph method to estimate the magnitude of discharges for design of drainage structures. Typically, Natural Resources Conservation Service (NRCS) methods are used. However, national NRCS methods have not been calibrated for Texas watersheds. Therefore, a review of the applicability of NRCS methods to Texas hydrology is appropriate.

The purpose of this report is to document results of a review of the technical literature dealing with unit hydrographs.

Background

The unit hydrograph theory is based on linear systems theory (Chow, *et al*, 1988). In the 1970's, Chow and others worked on development of linear systems theory applications to hydrologic modeling. Chapter 7 in Chow, *et al* (1988) is an overview of that work. The convolution integral,

$$Q(t) = \int_0^t I(\tau)(t - \tau)d\tau, \tag{1}$$

where: $Q(t)$ = output time function,
 $I(\tau)$ = input time function,
 $(t - \tau)$ = time lag between time the impulse is applied and t , and
 t = time.

In discrete time, the pulse response function is

$$Q_n = \sum_{m=1}^{n \leq M} P_m U_{n-m+1} \tag{2}$$

where: U_n = unit response function (unitgraph; L^2/T), and
 P_m = effective precipitation (L) for period m .

The unit hydrograph, then, is a simple linear model that has some embedded assumptions. They are:

- Effective rainfall has a constant intensity within the effective duration,
- Effective rainfall has a uniform spatial distribution,
- Time base of runoff (period of time that direct runoff exceeds zero) resulting from an effective rainfall of specific duration is constant,
- The ordinates of direct runoff of a constant base time are directly proportional to the total amount of direct runoff represented by each hydrograph, and
- For a particular watershed, the size of the direct runoff hydrograph for two effective rainfall pulses is in direct proportion to the relative size of the pulses.

In actuality, these assumptions are often not true, albeit difficult to test. The assumptions are particularly weak for small watersheds, which have a tendency to be non-linear in response. However, the unit hydrograph approach is usually good enough to obtain engineering estimates for design purposes.

The unit hydrograph, or unitgraph, is defined to be the hydrograph of runoff resulting from a unit pulse of runoff with a specific duration in time. Units of measurement vary with the particular procedure being applied, but in general, the depth of runoff is defined to be one unit (dimensionless) and the rate of runoff of the unitgraph has units of L^2/T .

Sherman (1932) developed the unit hydrograph procedure. Sherman recommended that the procedure should be used with watershed drainage areas that are less than about 2,000 square miles. If storm patterns are thought to impact runoff hydrographs, then the watershed can be subdivided into smaller sub-watersheds and each of those subjected to a hydrograph analysis. The development of the procedure has been documented many times.

Unit hydrographs are developed for a specific watershed using two basic approaches. If unit rainfall-runoff data are available, then numerous techniques can be applied to estimate a unit hydrograph from the measurements. If no data are available, then methods of synthetic hydrology must be applied. For the purposes of this report, attention will be focused on examination of methods for developing a unit hydrograph from measurements of rainfall and runoff.

Traditional Method

The traditional method for deriving a unit hydrograph uses an algorithm such as the following (Linsley, Kohler, and Paulhus, 1958).

- 1) Obtain time series of contemporaneous rainfall and streamflow measurements.
- 2) Separate the base flow from the hydrograph to produce the direct runoff hydrograph.

- 3) Compute the volume of runoff under the hydrograph by numerical integration (such as the trapezoid rule). When the beginning ordinate and ending ordinate are both zero, then the volume of runoff is given by

$$\frac{\sum Q\Delta t}{A}, \quad (3)$$

where Q is the instantaneous discharge, Δt is the time interval between ordinates, and A is the watershed drainage area.

- 4) Divide the ordinates of the DRH by the volume (depth) of direct runoff to obtain the unit hydrograph.
- 5) Determine the effective rainfall hyetograph by assuming that abstractions follow a uniform loss rate, called the *phi index*. The time duration of the pulses of effective precipitation (precipitation that exceeds abstractions) is the duration of the unit hydrograph.

Application of this procedure will yield one estimate of the unit hydrograph for each set of rainfall-runoff data. Clearly, runoff events with relatively uniform rainfall rates will produce better estimates of the unitgraph with this procedure. Furthermore, multiple applications of the algorithm are required to attain a sufficient number of unit hydrographs to assure a reasonable estimate for the watershed unit hydrograph. Finally, each instance of the unitgraph so derived will probably have a different duration. Therefore, lagging or S-hydrograph technology will have to be applied to bring all unitgraphs derived using the traditional procedure to the same duration.

Once a set of unitgraphs has been derived for a particular watershed and brought to the same duration, then they can be “averaged” to produce an estimate of the most reasonable unitgraph for a particular watershed.

Deconvolution of Rainfall-Runoff Events

Deconvolution is the process of extracting the unit response function (the unit hydrograph) from a direct runoff hydrograph and the generating precipitation sequence. The process is to solve Equation (2) for each U_n sequentially in a process that amounts to the back-substitution solution of a matrix equation. This first procedure is documented in Chapter 7 of Chow, *et al* (1988).

Collins (1939) developed a method of successive approximation that used all rainfall pulses to estimate a unit hydrograph. The procedure was:

- Assume a unit hydrograph and apply it to all effective rainfall blocks of the hyetograph except the largest;
- Subtract the resulting hydrograph from the actual direct runoff hydrograph and reduce the residual to unit hydrograph terms;
- Compute the weighted average of the assumed unit hydrograph and residual unit hydrograph and use that for the next trial; and
- Repeat the previous steps until the residual unit hydrograph does not differ by more than a permissible amount from the assumed hydrograph.

Problems with Collins' procedure are similar to those encountered with all of the deconvolution approaches. The resulting unit hydrograph may have erratic variations (high frequency oscillations) and may exhibit negative values. Negative values are disturbing in a physical sense as negative flow is implied.

Snyder (1956) applied the method of least squares to compute a unitgraph from observations of rainfall and runoff. He applied the method to ten storms from two different watersheds. Because effective rainfall was not known, he used a method of successive approximation to determine a constant loss rate to account for abstractions.

Nash (1959) studied the relation between the number of parameters (moments about the origin) and stream catchment characteristics for the instantaneous unit hydrograph.

Eagleson, *et al* (1966) examined unit hydrograph extraction from measured rainfall-runoff series. Use of a synthetic division (or back substitution, *a la* Chow, *et al* [1988]) approach resulted in significant oscillations in the underlying unitgraph. This occurs because of the structure of the matrices and because of the inherent lack of information in the rainfall-runoff series. So, as reported by Chow, *et al*, deconvolution will often result in unit hydrographs that have negative ordinates and high frequency oscillations, which are not physically realizable. So, the authors used the Weiner-Hopf equation and developed a linear programming solution to the convolution equation.

Colinearity refers to the linear relation between elements in a linear system (Bree, 1978). Because of the way that rainfall series are used to deconvolute the rainfall-runoff sequence, at least some degree of colinearity is present. The presence of this colinearity results in severe oscillations in the derived unitgraph (Delleur and Rao, 1971). Bree (1978) reported that colinearity, and hence the high-frequency oscillations could be reduced in derived unitgraphs by using multiple events in the deconvolution process.

Mawdsley and Tagg (1981) used the deconvolution method to derive unitgraphs from multiple events, similar to Bree (1978). Their experiments with multiple events led them to the same conclusions as Bree (1978).

Dooge and Bruen (1989) proposed use of the *condition number*, a linear algebra concept defined as square root of the ratio of the maximum eigenvalue to the minimum eigenvalue $P^T P$ matrix. Application of the condition number explains why some methods for unitgraph deconvolution have more stable behavior than other methods.

Bruen and Dooge (1992) re-examined the deconvolution process. For the least squares method,

$$X^T Xh = X^T y, \quad (4)$$

where: X^T = transpose of the effective precipitation matrix,
 X = effective precipitation matrix,
 h = unit hydrograph vector, and
 y = direct runoff vector.

Because the data used in evaluation of Equation (2) is prone to several kinds of errors, the results often do not conform to what an experienced hydrologist might expect. That is, a unit hydrograph is expected to have the following characteristics:

- Zero ordinates for all times prior to the start of precipitation excess;
- A runoff volume of unity in cases where the input and output volumes are equal (conservation);
- A smooth shape with a positive skew, no negative ordinates and no high frequency oscillations (a heavily damped system); and
- A unimodal shape typical of the majority of derived hydrographs.

When results deviate from these expectations, the deviations may be attributable to:

- Model error, when the catchment response is not approximately linear or time-invariant; and/or
- Errors in the data and possible amplification during the deconvolution.

The authors proposed use of ridge regression, in which an additional set of equations is added to the effective precipitation matrix,

$$(X^T X + kI)h = X^T y \quad (5)$$

where: k = a scalar parameter ($k \in [0,1]$), and
 I = the identity matrix.

Furthermore, assumptions about the shape of the unitgraph can be included in this analysis. The authors demonstrated application of this by including a shape assumption of a triangular unitgraph. Bruen and Dooge (1992) conclude that ridge regression represents a trade-off. Application of the procedure can produce unitgraphs with proper shape and stability, but at the cost of optimal linear fitting of the underlying data.

Linear Programming Estimates

Mays and Coles (1980) developed a linear programming approach to solving the unitgraph deconvolution problem. Mays and Taur (1982) extended this approach to a non-linear programming model that included estimation of rainfall abstractions. They concluded that, while the non-linear programming model offered some benefits in estimates of the unitgraph, but that the linear programming model was able to take on a larger number of events simultaneously. Unver and Mays (1984) revisited the non-linear programming model. They were able to attain estimates of loss-rate function and unitgraph, but reported that local optima may be the best solution available. This occurs because of errors in the rainfall-runoff observations and because some watersheds may not behave in line with the linear assumptions of the infiltration and unitgraph models.

Zhao and Tung (1994) reviewed application of linear programming techniques to unitgraph derivation using Bree's (1978) data set. They considered objective functions

applied by previous investigators, minimization of sum of absolute deviations and minimization of largest absolute deviation, and added two additional objective functions, minimization of weighted sum of absolute deviations and minimization of range of deviations. They reported that using the minimization of range of deviations objective function produced smoother unitgraphs and the reconstituted direct runoff hydrographs were better matches as well.

Other Studies

Commons (1942) developed a unit hydrograph for use in Texas.

Dooge (1959) established the basis for application of linear systems theory to hydrograph analysis. His study was the first to establish the theoretical basis for unit hydrographs. Previous studies focused on the empirical nature and application of the technology to solution of engineering problems.

Bender and Roberson (1961) developed a method for generating synthetic unitgraphs for Washington state watersheds. They based their general shape on the NRCS dimensionless unitgraph, and then generalized the shape to better fit unitgraphs taken from 19 study watersheds.

Holtan and Overton (1963) derive what appears to be the NRCS dimensionless hydrograph from data taken from 40 Midwestern and Eastern watersheds. They analyzed 70 hydrographs from these watersheds to determine a linear approach.

Croley (1980) discussed use of the two-parameter Gamma distribution for constructing a unit hydrograph. The two-parameter Gamma distribution has a positive skew and can be scaled to produce a unit runoff response. Therefore, it can be used to generate a unitgraph shape.

McCuen and Bondelid (1983) examined the synthetic NRCS dimensionless unit hydrograph. The peak discharge of the NRCS dimensionless unit hydrograph is given by

$$q_p = \frac{D_f A Q}{t_p}, \quad (6)$$

where: q_p = unit peak discharge (L^2/T),
 D_f = peak rate factor (units conversion, dimensionless),
 A = watershed drainage area (L^2),
 Q = watershed runoff depth (L), and
 t_p = time to unit peak discharge (T).

The standard peak factor is 484 (with units of square miles, hours, cfs/inch), although SCS (1972) indicates that D_f can vary from 300 to 600 depending on watershed conditions. McCuen and Bondelid used data from six watersheds to calibrate and test their method for adjusting the NRCS dimensionless unitgraph. They concluded that the

peak rate factor does impact runoff computation used for design activities. It is appropriate to calibrate the peak rate factor when data are available.

Boufadel (1998) fit the Nash unit hydrograph model to unitgraphs from five watersheds. He used two approaches to determine the Nash model parameters, the method of moments and constrained linear optimization. He found that the Nash model can provide a short-duration unitgraph from longer duration unitgraphs with some exceptions.

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