Optimal Detector Location For Bus Signal Priority

Hongchao Liu*
California PATH Program
Institute of Transportation Studies
University of California, Berkeley
Richmond Field Station, Bldg. 452
1357 S.46th Street
Richmond CA 94804-4648
Phone: (510)231-5725, Fax: (510)231-9512
E-mail:hongchao@path.berkeley.edu

Alexander Skabardonis
Wei-bin Zhang
Institute of Transportation Studies
University of California, Berkeley
109 McLaughlin Hall, Berkeley, CA 94720-1720
Phone: (510) 642-9166, Fax: (510) 642-1246
E-mail: skabardonis@ce.berkeley.edu

Meng Li
Institute of Transportation Studies
University of California, Berkeley
109 McLaughlin Hall, Berkeley, CA 94720-1720
Phone: (510) 643-4149, Fax: (510) 643-5456
E-mail: meng_lee@uclink.berkeley.edu

For Publication
Transportation Research Record
April 2004
Washington, D.C.

March 15, 2004
No. WORDS: 3,564
ABSTRACT

This paper presents a theoretical model to quantitatively address the relation between bus detector location and effectiveness of transit signal priority (TSP) systems, supplemented by a sample model application and simulation experiments on a real-life arterial corridor. The simulation results agree very well with results from the theoretical model. The findings of this research contribute to a better understanding of the interaction of the various components of TSP systems that can lead to better TSP design and implementation.

KEYWORDS

Transit Signal Priority, Bus Detector Location
INTRODUCTION

The objective of transit signal priority (TSP) systems is to provide priority to transit vehicles at signalized intersections while minimizing the adverse impacts to the rest of the traffic stream. Existing TSP systems detect the presence of a bus approaching the intersection and place a priority request on the downstream traffic signal. The signal responds to the priority call immediately, subject to safety and operational constraints, by either extending the current green phase (Green Extension), or truncating the current red phase(s) to serve the requested phase (Early Green). The effectiveness of TSP depends on the bus detection system because unnecessary priority requests do not improve the bus travel times and at the same time significantly increase the delay to the other traffic by disrupting normal signal control operations.

TSP detection systems fall into two main categories: Selective Vehicle Detection (SVD) or Automatic Vehicle Location (AVL). AVL systems provide continuous monitoring of bus location and have shown promise as detection means for TSP, but most of the existing TSP implementations use SVD systems with radio frequency tag/reader and inductive loops (1). Examples include Seattle’s Amtech system (2), Los Angeles’s Loopcom system (3), and Portland’s Opticom system (4).

Literature review on existing TSP systems indicates that there are no clear guidelines on bus detector placement. Reported locations range from 300 ft (100 m) to 1500 ft (500 m) upstream of the intersection stopline (5, 6). Such big difference may lead to quite different signal priority performance. Bus detectors located too close to the intersection trigger traffic signal for priority service that may be too late to truncate the other phases to provide the early green interval. Placing the detector too far upstream may result in failure to discharge the bus during the green extension interval, or to request unnecessary priority service when the bus can clear the intersection during the normal green phase. There is a need to quantitatively address the relationships between bus detector location and effectiveness of TSP.

The objective of the study described in this paper is to investigate the relationship between bus detector location and effectiveness of TSP systems, and to develop a model to calculate the optimal bus detector location. The study is part of larger research project to design and field test adaptive TSP systems on a real life corridor (7).

The next section of the paper presents the formulation of the theoretical model and its application. The following section presents the simulation experiments performed on a real life arterial and the analysis of the results. The last section summarizes the study findings along with recommendations for future research.
RESEARCH APPROACH

Theoretical Analysis

The bus travel time from the detection point to the intersection stopline is a function of traffic demand, queue length ahead and current signal timing if there is no bus stop in between; furthermore, the signal timing is a function of traffic demand in actuated signal control systems. The interaction between signal timing and traffic flow makes the modeling approach rather arduous especially under oversaturated flow condition. We assume (1) the intersection is undersaturated and queuing vehicles on each approach clear within one cycle without residual queue left on the beginning of the next cycle; (2) the actuated traffic signals operate as fixed time with constant green times, when the degree of saturation is higher than a certain value; (3) signal priority is provided by early green or green extension subject to the constraints of minimum green and maximum green extension thresholds.

Subject to the first assumption, the principle of point queue applies. Point queue is the vertical queue behind the stopline without physical length; a vehicle travels the link at free flow speed and enters the end of the vertical queue. As bus free-flow travel time is constant, the bus arrival time to the intersection depends to the time needed to clear the queue on the bus approach. When a bus is detected (and requests priority) at time $t$, the time needed to clear the queue in front of the bus is composed of the waiting time for the green light and the time to discharge the queue:

$$ T(t) = T_1(t) + T_2(t) $$

(1)

where:

$T(t)$ = Time needed to clear up the queue at time $t$

$T_1(t)$ = Waiting time for the green phase

$T_2(t)$ = Time to clear the queue during the green phase

We use the following notation:

$N$ = Number of signal phases;

$k$ = Current phase at time $t$, $k = 1, 2...n$;

$j$ = The Requested phase for signal priority, $j = 1, 2...n$;

$MinG_i$ = Guaranteed minimum green interval for phase $i$, which is the minimum green interval of the phase $i$ plus the yellow and all-red time;

$G_i$ = Green time for phase $i$ (Average green time for the actuated phase $i$ plus the yellow and all-red lost time);

$MaxExt$ = Maximum allowable interval for green extension;

$\lambda_j$ = Traffic arrival rate for phase $j$;
\[ N_Q(t) = \text{Number of queued vehicles in front of bus at time } t; \]

\[ TT_{ff} = \text{Bus free flow travel time from bus detector to stop line;} \]

\[ S_j = \text{Saturation flow for phase } j. \]

We partition the local signal cycle according to the designated signal phasing, and map the detection time of bus, \( t \), into the cycle as shown in Figure 1. Suppose the bus is detected at time \( t \) on phase \( k \) and requesting priority for phase \( j \). The times \( T_1 \) and \( T_2 \) can be estimated as follows:

\[
T_1(t) = \left\{ \begin{array}{ll}
\sum_{i=k+1}^{i-1} \text{Min}G_i + i \sum_{i=1}^{i-1} G_i + \text{Min}G_k - t \times \gamma(t) & (k < j) \\
\beta(t) \times \sum_{i=1}^{i-1} G_i - t + \text{MaxExt} + \sum_{i=j+1}^{n} \text{Min}G_i + \sum_{i=1}^{i-1} \text{Min}G_i & (k = j) \\
\sum_{i=k+1}^{i-1} \text{Min}G_i + (i \sum_{i=1}^{i-1} G_i + \text{Min}G_k - t) \times \gamma(t) + \sum_{i=1}^{i-1} \text{Min}G_i & (k > j)
\end{array} \right.
\]

\[
N_Q(t) = \left\{ \begin{array}{ll}
\lambda_j \times (t + \sum_{i=j+1}^{n} G_i) & (k < j) \\
\max(0, \beta(t) \lambda_j (TT_{ff} - \text{MaxExt} - \sum_{i=1}^{i-1} G_i + t), \lambda_j t - S_j (t - \sum_{i=1}^{i-1} G_i)) & (k = j) \\
\lambda_j \times (t - \sum_{i=1}^{i-1} G_i) & (k > j)
\end{array} \right.
\]

\[
\gamma(t) = \left\{ \begin{array}{ll}
1; & (0 \leq t < \sum_{i=1}^{i-1} G_i + \text{Min}G_k) \\
0; & (\sum_{i=1}^{i-1} G_i + \text{Min}G_k \leq t < C)
\end{array} \right.
\]

\[
(4)
\]

\[
\beta(t) = \left\{ \begin{array}{ll}
1; & (\sum_{i=1}^{i-1} G_i - TT_{ff} + \text{MaxExt} < t < C)
\end{array} \right.
\]
In the case \( k = j \), the green extension applies and the maximum allowable extended interval is \( \text{MaxExt} \). Otherwise, the signal provides early green by terminating the normal green time at the end of \( \text{MinG}_i \) of each phase \( i \) prior to the requested phase \( j \). Equations (2) and (3) calculate \( T_j(t) \) and \( N_Q(t) \) according to the position of time \( t \) on the local signal cycle.

The dummy variable \( \gamma(t) \) equals to 1 when \( t \) is prior to the termination of the guaranteed minimum green of the current phase \( k \), \( \text{MinG}_k \). As the green extension is initiated at the end of phase only if the check-out call has not been received during the normal green interval, if the requesting time \( t \) mapped in the signal cycle is beyond the threshold defined in Equation (5), the bus cannot pass the intersection within the \( \text{MaxExt} \) and \( \beta(t) \) is set to 1, otherwise \( \beta(t) \) equals to 0.

Both \( T_j(t) \) and \( T_z(t) \) can be seen as random events, statistical analysis is the appropriate approach to find the best solution of the location of bus detectors. We assume that the bus arrival times at the bus detectors are uniformly distributed on the local signal cycle. The expectations of \( T_1, T_2 \), and \( T \) can be calculated by:

\[
E(T_1) = \sum_{i=0}^{j-1} G_i \int_{t=0}^{T_1(t)} f(t) dt + \int_{t=\sum_{i=0}^{j-1} G_i}^{\text{MaxExt}} T_1(t) f(t) dt + \int_{t=\sum_{i=0}^{j-1} G_i}^{C} T_1(t) f(t) dt
\]

(6)

\[
T_z(t) = N_Q(t) / S_j
\]

(7)

\[
E(T_2) = \sum_{i=0}^{j-1} G_i \int_{t=0}^{N_Q(t) / S_j} f(t) dt + \int_{t=\sum_{i=0}^{j-1} G_i}^{N_Q(t) / S_j} N_Q(t) f(t) dt + \int_{t=\sum_{i=0}^{j-1} G_i}^{C} N_Q(t) f(t) dt / S_j
\]

(8)
\[ E(T) = E(T_1) + E(T_2) \]  
\[ S_{\text{det}} = v_{ff} \times E(T) \]

where,
\[ f(t) = \text{Probability Distribution Function (PDF) of } t; \]
\[ C = \text{Cycle length}; \]
\[ S_{\text{det}} = \text{Statistical optimal location for bus detector}; \]
\[ v_{ff} = \text{Bus free flow speed}. \]

**Application of the Proposed Methodology**

The methodology was applied to a section of El Camino Real/California State Highway 82, a major arterial in San Francisco Bay Area (Figure 2). The study section includes four signalized intersections. The intersection spacing varies from 510 ft (170 m) to 1740 ft (580 m). The traffic signals are equipped with fully actuated Type 170 controllers. There are no nearside bus stops at the test section.

Traffic signals operate as coordinated/semi-actuated with a common cycle length of 90 seconds during the am peak analysis period. The typical phase sequence is lead-lag left turn phases on El Camino Real (phases 1&5) with synchronization on phase 2 (through movement on the southbound El Camino Real).

The proposed methodology was applied on the southbound direction at the intersections of 20th Ave., 25th Ave., and 27th Ave. Table 1 shows the basic input data to the model including number of phases, phase sequence, minimum green interval, green interval, cycle, traffic demand on each phase, saturation flow and bus free-flow speed. In accordance with the fixed-time control assumption in the theoretical model, the green times per phase is the average green interval in the analysis period. Traffic demands are the average flows during the morning peak period from 7:45 am to 8:45 am.

The optimal locations are computed based on the statistical expectation of \( T_1(t) \) and \( T_2(t) \); Figure 3 shows graphically the relationship of the bus arrival time to the time waiting for the green light and the time waiting for discharging queue at the intersection of El Camino & 20th Avenue. The calculated optimal bus detector locations are 580 ft (175 m), 720 ft (217 m), and 425 ft (128 m) upstream from the intersections of 20th Ave., 25th Ave., and 27th Ave. respectively.

**Simulation Experiments**
A basic assumption of the theoretical model is that traffic signals operate as fixed-time signal in which each phase is given a constant interval. However, most traffic signals on real-life arterials are actuated signals, where the interval of a phase is actually the function of traffic demand. Also, the theoretical approach could not describe the impacts of different detector location strategies on the other traffic. To further address these issues, simulation analysis is introduced in this section.

The simulation tool used in this research was developed by the authors specifically for the analysis of TSP systems (8). The tool, dubbed Paramics/TSP, was developed through Application Programming Interface (API) on Paramics, a commercial microscopic traffic simulation software produced by Quadstone Ltd (9). It models a NEMA or Type 170 eight-phase dual-ring fully actuated signal controller. The coordination API is developed based on the full-actuated signal control logic, with additional force-off and logic to maintain the background cycle and the yield point logic for synchronization.

The model allows the user to define all the functions of the proposed TSP system to be simulated. In this study, the TSP was defined as follows:

- Signal priority applies to arterial through phases 2 and 6;
- No signal priority at the signalized intersections with nearside bus stops;
- One priority service in every two signal cycles, the cycle following the TSP cycle is set as transition cycle;
- Early green applies when current phase is red at the time of bus detection, otherwise, green extension applies;
- Green extension initiates at the end of the phase when check-out call has not been received during normal green, check-out detectors are placed at the entrance of downstream link. Green extension terminates at the early time point of the check-out detection and the maximum extension;
- Early Green initiates instantaneously upon the detection of bus;
- Maximum green extension and minimum green interval are same as defined in the theoretical model.

The analysis of the simulation results focused on the intersection delay of busses and other traffic. The simulation records the times 1) when a bus is released to the network; 2) when a bus stops at a signalized intersection; 3) when a bus stops at a bus stop; 4) when a bus leaves a bus stop; 5) when a bus passes the stop line of an intersection; 6) when a bus ends its trip. The intersection delay for a bus is defined as the time difference between when the vehicle stops at the traffic signal and the time it crosses the intersection stopline. A vehicle is defined as ‘stopped’ in the simulation if the velocity is less than 3 mph (5 kph).

The analysis of the simulation results focused on the intersection of El Camino/25th Avenue. Four traffic and control scenarios were simulated, all with bus frequency of 10 minutes (6 busses/hr):

1. Existing conditions (degree of saturation = 0.72, cycle length = 90 sec)
2. Higher traffic demand and same signal timing (degree of saturation = 0.80, 0.90; cycle length = 90 sec)
(3) Lower traffic demand and same signal timing (degree of saturation = 0.50, 0.60; cycle length = 90 sec)

(4) Higher traffic demand with longer cycle length (degree of saturation = 0.90, cycle length = 110 sec)

Traffic demand and signal timing in scenario (1) are the same to the inputs used in the sample calculations using the theoretical model. The scenarios two, three, and four are designed to conduct sensitivity analysis of the optimal location to traffic demand and signal cycle. The saturation degree of the target intersection is increased approximately to 0.8, 0.90 in the scenario two and decreased to about 0.6 and 0.5 in the scenario three by adjusting the origin and destination flows. In the scenario four, signal cycle is enlarged to 110 seconds, other signal parameters such as minimum green, maximum extension are changed accordingly.

Initial simulation runs are performed under the definitions of the scenario one but without signal priority. The average delay of non-transit vehicles at the intersection of 25th Ave. is 12.4 sec/veh for the through movements on the El Camino Real and 34.8 sec/veh for all movements on the cross street, the average bus delay is 14.7 sec/bus. Then, under each scenario, the bus detector is moved gradually from 300 ft (100 m) to 1500 ft (500 m) away from the intersection of 25th Ave. with a step increment of 300 ft (100 m). Five simulation runs are executed with respect to each location under the same scenario, a total of 25 runs are performed for each scenario.

Figure 4(1) depicts the simulation result of the case one, which reveals a clear relation between the location of bus detector and bus intersection delay. Additional simulation runs are made with a step increment of 150 ft (50 m) when simulation showed the best location fell into the area of between 450 ft (150 m) and 750 ft (250 m) from the intersection, which further give the best solution of 600 ft (200 m) to the target intersection. The simulation result shows the optimal solution agrees with the result of the calculation, which is 720 ft (217 m), very well.

Simulation tests of the scenario two with higher traffic demand still shows a well relation, as shown in Figure 4(2), with the optimal point shifted about 150 ft (50 m) to the upstream of the intersection. For degrees of saturation 0.60 and lower, the relation does not follow a concave curve with a unique point of minimum bus delay but instead presents a declining curve with multiple optimal solutions, which is illustrated in Figure 4(3). Figure 4(4) depicts the simulation result of scenario four, which shows the relation is still tenable and gives 1050 ft (350 m) to the intersection the optimal location.

The intersection delay of other vehicles was calculated separately for the through and left turn movements on the El Camino Real arterial and as a total on the cross street. Figure 5 illustrates the relationship of delays of other vehicles and the bus detector location for each scenario. Signal priority increases the delay on the cross streets and the left turn movements, especially for degrees of saturation higher than 0.80 and for the higher cycle length.

The randomness of simulation is considered by conducting five simulation runs for each scenario and taking the average value as the result. Statistical test is conducted to verify the simulation results. Figure 6 shows the distributions of bus intersection delay resulted from the simulation scenario one. All the distribution curves for different bus detector positions
are unimodal. The Z-test and F-test presented in Table 2 proves the optimal location is 600 ft (200 m) to the intersection.

**DISCUSSION**

This paper presents the methodology and findings of a study to quantitatively address the relationship between the bus detector location and effectiveness of transit signal priority (TSP) systems. A theoretical model was developed to predict the bus travel time to the intersection and to calculate the optimal detector location. Simulation experiments were performed on a real-life corridor to validate the results of the theoretical model and estimate the impacts of bus detector location for a range of traffic demand and signal control scenarios.

The results indicate that performance of bus signal priority is, to a considerable extent, related to the location of bus detectors. Although the optimal location is a function of traffic demand and signal timing, the results show that placing detector between 450 ft (150 m) and 900 ft (300 m) upstream of the intersection produces good results for most of the conditions analyzed in this study.

Traffic patterns may change over time and may necessitate changes to the detector placement. This may be difficult for existing detector installations. A possible solution is to design a ‘delay timer’ in traffic signal controllers that is able to ‘hold’ the signal priority requests for a flexible period of time. The designs of such schemes as well as other aspects of TSP are the subject of ongoing research.
ACKNOWLEDGMENTS

This study was performed as part of the California's PATH (Partners for Advanced Highways and Transit) Program at the Institute of Transportation Studies (ITS) University of California Berkeley. We are grateful to Sonja Sun, Kai Leung of New Technology California Department of Transportation (Caltrans) Headquarters, and Paul Chiu of Caltrans District 4 for their guidance and support throughout the project. We appreciate TJKM Transportation Consultants for providing the traffic data used in the study.

The contents of this paper reflect the views of the authors who are responsible for the facts and the accuracy of the data presented herein. The contents do not necessarily reflect the official views of or policy of the California Department of Transportation. This paper does not constitute a standard, specification or regulation.
REFERENCES


LIST OF FIGURES

FIGURE 1  Mapped $t$ on Local Signal Cycle
FIGURE 2  The Test Site for Application and Simulation
FIGURE 3  Bus Arrival Time Vs. $T_1$ and $T_2$
FIGURE 4  Detector Location Vs. Bus Intersection Delay
FIGURE 5  Impacts On Other Traffic
FIGURE 6  Distribution Of Bus Intersection Delay (Scenario 1)
FIGURE 1 Mapped $t$ on Local Signal Cycle
FIGURE 2  The Test Site for Application and Simulation
FIGURE 3  Bus Arrival Time vs. $T_1$ and $T_2$
(1) Basic Scenario (Existing Conditions)            (2) High Demand

(3) Low Demand                     (4) Long Cycle Length

FIGURE 4 Detector location vs. Bus Intersection Delay
FIGURE 5  Impact on Other Traffic

(1) Basic Scenario  
(2) High Demand  
(3) Low Demand  
(4) Long Cycle Length
FIGURE 6 Distribution of Bus Intersection delay (Scenario 1)
LIST OF TABLES

TABLE 1. Sample Application Inputs

TABLE 2. Statistic Test
### TABLE 1 Sample Application Inputs

<table>
<thead>
<tr>
<th>Intersection</th>
<th>Signal phases&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Phase sequence</th>
<th>Minimum green(sec)</th>
<th>Green (sec)</th>
<th>Lost time (sec)</th>
<th>Cycle (sec)</th>
<th>Demand (veh/hr.)</th>
<th>Saturation flow(veh/hr/lane)</th>
<th>Number of lanes</th>
<th>Bus speed&lt;sup&gt;b&lt;/sup&gt; (mile/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(lead)</td>
<td>3</td>
<td>6</td>
<td>16</td>
<td>18</td>
<td>198</td>
<td>1620</td>
<td>1</td>
<td>1</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>20th Ave.</td>
<td>2</td>
<td>4</td>
<td>15</td>
<td>32</td>
<td>1668</td>
<td>1800</td>
<td>1</td>
<td>1</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>6</td>
<td>16</td>
<td>12</td>
<td>215</td>
<td>1620</td>
<td>1</td>
<td>1</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>5(lag)</td>
<td>4</td>
<td>6</td>
<td>14</td>
<td>16</td>
<td>162</td>
<td>1800</td>
<td>1</td>
<td>1</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>15</td>
<td>32</td>
<td>16</td>
<td>135</td>
<td>1620</td>
<td>1</td>
<td>1</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>8</td>
<td>16</td>
<td>14</td>
<td>115</td>
<td>1620</td>
<td>1</td>
<td>1</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>8</td>
<td>18</td>
<td>18</td>
<td>1656</td>
<td>1800</td>
<td>3</td>
<td>1</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>25th Ave.</td>
<td>4</td>
<td>1</td>
<td>8</td>
<td>18</td>
<td>206</td>
<td>1800</td>
<td>1</td>
<td>1</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>5(lag)</td>
<td>4</td>
<td>8</td>
<td>14</td>
<td>12</td>
<td>101</td>
<td>1620</td>
<td>1</td>
<td>1</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>10</td>
<td>30</td>
<td>12</td>
<td>1632</td>
<td>1800</td>
<td>3</td>
<td>1</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>8</td>
<td>16</td>
<td>10</td>
<td>221</td>
<td>1620</td>
<td>1</td>
<td>1</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>8</td>
<td>18</td>
<td>8</td>
<td>245</td>
<td>1800</td>
<td>1</td>
<td>1</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>27th Ave.</td>
<td>5(lag)</td>
<td>3</td>
<td>8</td>
<td>18</td>
<td>9</td>
<td>200</td>
<td>1620</td>
<td>1</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>10</td>
<td>39</td>
<td>9</td>
<td>1650</td>
<td>1800</td>
<td>3</td>
<td>1</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>8</td>
<td>22</td>
<td>8</td>
<td>210</td>
<td>1800</td>
<td>1</td>
<td>1</td>
<td>30</td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup> phase 2 is the requested phase for signal priority

<sup>b</sup> free flow speed
### TABLE 2 Statistic Test

<table>
<thead>
<tr>
<th>Bus detector location</th>
<th>100</th>
<th>200</th>
<th>250</th>
<th>300</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of buses</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>Mean (sec)</td>
<td>8.89</td>
<td>6.78</td>
<td>4.83</td>
<td>7.34</td>
</tr>
<tr>
<td>Standard Error (sec)</td>
<td>10.40</td>
<td>7.71</td>
<td>8.29</td>
<td>14.50</td>
</tr>
</tbody>
</table>

**Z-test**

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Test 1</th>
<th>Test 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_{200} &gt; \mu_{250} )</td>
<td>Not Reject</td>
<td>Not Reject</td>
</tr>
<tr>
<td>( \mu_{250} &lt; \mu_{300} )</td>
<td>Not Reject</td>
<td>Not Reject</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>( s^2_p )</td>
<td>64.0841</td>
<td>139.48705</td>
</tr>
<tr>
<td>Z-statistic</td>
<td>2.310898</td>
<td>-2.016174</td>
</tr>
<tr>
<td>Z-value</td>
<td>1.64</td>
<td>-1.64</td>
</tr>
<tr>
<td>Conclusion</td>
<td>Not Reject</td>
<td>Not Reject</td>
</tr>
</tbody>
</table>

**F-test**

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Test 1</th>
<th>Test 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma^2_{100} &gt; \sigma^2_{200} )</td>
<td>Not Reject</td>
<td>Not Reject</td>
</tr>
<tr>
<td>( \sigma^2_{200} &lt; \sigma^2_{250} )</td>
<td>Not Reject</td>
<td>Not Reject</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>F-statistic</td>
<td>1.819525</td>
<td>0.8649673</td>
</tr>
<tr>
<td>F-value</td>
<td>1.35</td>
<td>1.35</td>
</tr>
<tr>
<td>Conclusion</td>
<td>Not Reject</td>
<td>Not Reject</td>
</tr>
</tbody>
</table>