Entropy-based Estimation of Transfers in a Terminal

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Abstract: An intermodal transportation terminal is a facility that provides commuters with easy transfer between many transit modes and providers such as buses, light rail, subway, taxis, airport shuttles, and commuter rails. The probability of a passenger transferring from one mode to another and the estimation of total transfer demand are of great importance to both practitioners and researchers when determining the optimal design alternatives as well as the best control and management policies for daily operation of the terminal. This paper presents a study that uses an entropy-based optimization approach to estimate the transfer demands between the available transportation modes in an intermodal transportation terminal. The development and calibration of the entropy model is presented in the first part of the article, which is followed by a case study at the Sihui Intermodal Terminal in Beijing, China. The study contributes to the literature and practice by offering a real world experience in transfer probability estimation at intermodal terminals.

CE Database subject headings: Transfer Demand; Intermodal Transportation Terminal; Entropy Model.
Introduction

Multimodal transportation terminals are important nodes in a transportation network that play a pivotal role in overall performance of the transportation system. Design, operation, and management of such terminals rely largely on the information of transfer demands among the available transportation modes. This paper presents an entropy based modeling approach that estimates the transfer demands at urban transportation terminals, as well as its application to a transfer terminal in Beijing, China.

A multimodal transportation terminal is a facility that provides commuters with easy transfer between many transit modes and providers such as buses, light rail, subway, taxis, airport shuttles, and commuter rails. The probability of a passenger transferring from one mode to another is of great interest to both practitioners and researchers when determining the optimal design alternatives as well as the best control and management policies of a transportation terminal. However, very limited information was found in the literature that specifically addresses this problem. Bookbinder and Desilets (1992) developed a combined optimization and simulation approach to minimize the overall inconvenience to passengers transferring between lines in a transit network; similar research can be found in Hall et al. (2001) which attempts to optimize bus holding times at transfer stations, and in the work of Ting and Schonfeld (2005) which developed a coordination model for transit scheduling in multimodal transportation hubs. Though relevant, the objective of these studies was to find best bus scheduling policies rather than estimating the transfer demand. Knoppers and Muller (1995) touched on the problem by investigating the possibilities and limitations of coordinated transfers in public transit. The objective of their study was extended to minimizing passenger’s transfer time in a multimodal transportation system.

One of the challenges in estimating transfer demands is selecting a proper methodological approach that is not only theoretically sound, but also of practical values for real-life application (Kikuchi and Tyler, 2005). Gravity models, Entropy models, network modeling, and the four-step planning process within the regional/metropolitan multimodal travel demand forecasting framework could all possibly be used to the problem.

As a fundamental node in a multimodal transportation network, a transfer terminal plays an important role in the system through its impact to a traveler’s travel time. It is thus feasible for a network modeling approach which is aimed to minimize the total travel time and takes the transfer time in the terminal as a variable (Duff-Riddell and Bester, 2005). The issue of concern for a network based approach, however, lies in its limitation in data requirements (e.g., Origin-Destination data must be obtained from individual users) and computational efficiency, and thus its practical value in presenting multimodal transportation networks. On the other hand, a full planning process that goes beyond the terminal itself and takes into account the evolving regional OD and travel patterns would be extremely time consuming and over dependent to the demographic data that are sometimes not available.
Another feasible approach is to use spatial interaction models to describe the dispersion and coherence of the transferring activities in a transfer terminal, which is a typical spatial system. Gravity and entropy theories fall into this category. Gravity theory describes the degree of spatial interaction between two or more points in a spatial system in a manner analogous to point to point movements in a physical system (Sasaki, 1968; Nijkamp, 1978; Wang et al., 2006). The entropy maximization models, which originally came from thermodynamics, present the traditional gravity interaction models in a probabilistic framework (Betty, 1972). Application of gravity and entropy-maximizing models in transportation can be found in numerous studies related to transportation planning, especially trip distribution models (e.g., Wilson, 1970; Beckman and Golob, 1971; Cesario, 1973; Fisk, 1988; Leung and Yan, 1997, and Shao et al., 2006).

The initial motive of this research was to use the spatial interaction approach to model transfer demands between given transportation modes in a multimodal transportation terminal. Both gravity model and entropy model were developed and examined against real field data. The attractiveness and transfer resistance of each transportation mode are defined based on its speed, transfer time, cost, and level of comfort. The objective of the gravity model and the entropy-maximizing model is to find the distribution matrix of the transfer demands between each transportation mode. In the case study, a total of seven transportation modes are considered, which includes bicycle, walk, city bus, suburban bus, passenger car, subway, and taxi. The relevant data was either obtained from the Beijing Municipal Commission of Urban Planning or field survey of passengers. The focus of this article is to introduce our study on the entropy-maximizing approach with the gravity prior probability.

The remainder of the paper is structured as follows. Section 2 introduces the factors influencing the transfer mode choice and the proposed entropy-maximizing model with Gravity prior probability for estimating the transfer demand matrices. Section 3 presents the application of the model through a case study conducted in Beijing. A comparison between the Entropy model and the Gravity model is presented in Section 4, followed by the conclusions presented in Section 5.

The Entropy-maximizing Approach for Transfer Demand Estimation

Factors Influencing a Passenger’s Mode Choice

A multimodal transportation terminal is a unique node in the transportation network which plays a significant role in the system in terms of its impact to passenger’s total travel time. A passenger’s mode choice at the terminal is affected by many factors, among which the carrier’s speed, service frequency,
level of comfort, cost, and accessibility are arguably most influential factors that must be considered in the forecasting model.

The average operating speed is probably the most important factor when a traveler selects a specific mode of transportation as it directly relates to the travel time of the entire trip. The frequency of service of a connecting transportation mode in the terminal is another determinant factor, which affects the mode choice in the form of waiting time within the terminal. The level of comfort, fare, and accessibility of the transfer station also influence, to varying degrees, the mode choice and thus transfer demands. In addition, different life styles may also affect how people decide to use their resources for transportation. For instance, a certain group of travelers may be more sensitive to the quality of service (e.g., retired people) than another group whose primary priority might be the operating speed.

It is a challenging endeavor to model transfer demands when there are so many factors that come into play, especially when considering that all these factors affect a traveler’s transfer behavior differently. In order to carry out a demand forecasting process, the determining factors need to be identified and presented in a quantitative manner in the context of their degree of importance to mode choice behavior at the terminals. A survey of nearly 2000 passengers was conducted in 2008 to identify the key factors as well as their relative importance to mode choice at transfer terminals. According to the survey, speed, service frequency, fare, and level of comfort of a transportation mode are the four main factors that predominantly influence a passenger’s decision for mode choice. The relative importance of these factors was also obtained, according to which the weighting coefficient of each factor was calculated. Details of these data are discussed along with the experimental study.

**Modeling Transfer Demands in an Entropy-maximizing Approach**

With the influencing factors for mode choice discussed, attention can now be directed to the modeling approach. In the model, the probability for a traveler transferring from mode $i$ to other modes is denoted by $f_i$. The probability for a traveler transferring from other modes to mode $j$ is denoted by $g_j$. The probability for a traveler transferring from mode $i$ to $j$ is denoted by $h_{ij}$. They can be calculated as follows.

\[
f_i = X_i / X, \quad \left(\sum_i f_i = 1\right)
\]

\[
g_j = Y_j / X, \quad \left(\sum_j g_j = 1\right)
\]

\[
h_{ij} = X_{ij} / X_i, \quad \left(\sum_j h_{ij} = 1\right)
\]
Where, \( X_i \) is the transfer demand generated from mode \( i \). \( Y_j \) is the transfer demand attracted to mode \( j \), and \( X_{ij} \) is the transfer demand from mode \( i \) to mode \( j \). \( X \) is the total transfer demand.

The conservation law reads,

\[
\sum_j h_{ij} = 1 \tag{4}
\]

\[
\sum_i f_i h_{ij} = g_j \tag{5}
\]

In accordance with the Gravity model, the prior probability is

\[
q_{ij} = \alpha f_{ij} g_j t_{ij}^\gamma \tag{6}
\]

Where, \( q_{ij} \) is the gravity prior probability of the transfer demand from mode \( i \) to mode \( j \). \( t_{ij} \) is the travel expense. \( \alpha \) and \( \gamma \) are regression parameters which need to be calibrated using the observed data.

Considering \( q_{ij} \), the joint probability distribution can be described as:

\[
F = \frac{X!}{\prod_i \prod_j X_{ij}!} \prod_i \prod_j (q_{ij})^{X_{ij}} \tag{7}
\]

The distribution deduced from the maximal \( F \) is of interest. The problem to maximize \( F \) is equivalent to the problem to maximize log-transformed \( F \), which could be expressed as:

\[
\ln F = \ln \left( \frac{X!}{\prod_i \prod_j X_{ij}!} \prod_i \prod_j (q_{ij})^{X_{ij}} \right)
\]

\[
= \ln(X!) - \sum_i \sum_j \ln(X_{ij}!) + \sum_i \sum_j X_{ij} \ln(q_{ij}) \tag{8}
\]

Using the Stirling’s Formula \( \ln(n!) \approx n \ln n - n \) and Eq. (6), we have:

\[
\ln F = X \ln X - X - \sum_i \sum_j (X_{ij} \ln(X_{ij}) - X_{ij}) + \sum_i \sum_j X_{ij} \ln(\alpha f_{ij} g_j t_{ij}^{\gamma}) \tag{9}
\]

Substituting \( \sum_i \sum_j X_{ij} = X \) into Eq. (9) to get

\[
\ln F = X \ln X - \sum_j \sum_i X_{ij} \ln(X_{ij}) + \sum_i \sum_j X_{ij} \ln(\alpha f_{ij} g_j t_{ij}^{\gamma})
\]

\[
= X(-\sum_i \sum_j f_i h_{ij} \ln h_{ij} - \gamma \sum_i \sum_j f_i h_{ij} \ln t_{ij}) + X \ln \alpha + X \sum_j g_j \ln g_j \tag{10}
\]

Since \( X, \alpha \) and \( g_j \) are constants, the objective function can be written as the following:

\[
R = -\sum_i \sum_j f_i h_{ij} \ln h_{ij} - \gamma \sum_i \sum_j f_i h_{ij} \ln t_{ij} \tag{11}
\]
Taking Eq. (4), (5) as the constraints, the problem can be expressed as follows:

\[
\begin{aligned}
R &= -\sum_i \sum_j f_i h_{ij} \ln h_{ij} - \gamma \sum_i \sum_j f_i h_{ij} \ln t_{ij} \\
\sum_j h_{ij} &= 1 \\
\sum_i f_i h_{ij} &= g_j
\end{aligned}
\]  

(12)

The formulation above is composed of one objective function and two constraints. The statement in this formulation indicates that the objective of this problem is to find the most likely ways to distribute transfer passengers. The first constraint suggests the total transfer probability from one node to all the other nodes should be equal to 1. The second constraint restricts the total transfer probability to a destination mode to be the sum of the probabilities of all origin modes making transfers to it.

Calibrating \( \gamma \) in Eq.(6) is essential to the Entropy-maximizing Model (EM Model) with gravity prior probability. The Log-transformed formula of Eq. (6) reads

\[
\ln q_{ij} = \ln \alpha + \ln f_i + \ln g_j - \gamma \ln t_{ij}
\]  

(13)

Where, \( q_{ij}, f_i, g_j, t_{ij} \) can be calculated from the observed transfer matrix and the characteristics of transportation modes. When there are \( n \) different transportation modes in the terminal, Eq. (13) can be expressed as

\[
y_k = \beta_0 + \beta_1 \ln f_i + \beta_2 \ln g_j + \beta_3 \ln t_{ij}, \quad k = 1, 2, \ldots, n^2
\]  

(14)

Where, \( \beta_0 = \ln \alpha, \beta_3 = -\gamma \), and

\[
y = \begin{bmatrix}
\ln q_{11} \\
\ln q_{12} \\
\vdots \\
\ln q_{n1} \\
\ln q_{n2} \\
\vdots \\
\ln q_{nn}
\end{bmatrix},
\quad
x = \begin{bmatrix}
1 & \ln f_i & \ln g_j & \ln t_{11} \\
1 & \ln f_i & \ln g_j & \ln t_{12} \\
\vdots & \vdots & \vdots & \vdots \\
1 & \ln f_i & \ln g_j & \ln t_{n1} \\
1 & \ln f_i & \ln g_j & \ln t_{n2} \\
\vdots & \vdots & \vdots & \vdots \\
1 & \ln f_i & \ln g_j & \ln t_{nn}
\end{bmatrix},
\quad
\beta = \begin{bmatrix}
\beta_0 \\
\beta_1 \\
\beta_2 \\
\beta_3
\end{bmatrix}
\]

\[
y = x \beta
\]  

(15)

As the result of Eq. (15), \( \beta \) is equal to \((x'x)^{1/2} x'y\). Therefore, \( \gamma = -\beta_3 \).

To find a maximal \( R \) under the constrains, the following Lagrange’s function is used.
\[ L = R + \sum_j \lambda_j (\sum_i f_i h_{ij} - g_j) + \sum_i \mu_i (\sum_j h_{ij} - 1) \]  

(16)

Where \( \lambda_j \) and \( \mu_i \) are indeterminate coefficients. Because,

\[
\begin{align*}
\frac{\partial L}{\partial h_{ij}} &= 0 \\
\frac{\partial L}{\partial \lambda_j} &= 0 \\
\frac{\partial L}{\partial \mu_i} &= 0
\end{align*}
\]

We have

\[
\begin{align*}
\lambda_j &= \ln g_j - \ln [\sum_i f_i \exp(-1 + \frac{\mu_i}{f_i} + \lambda_j) t_{ij}^{-\gamma}] \\
\mu_i &= -f_i \ln [\sum_j \exp(-1 + \lambda_j) t_{ij}^{-\gamma}] \\
h_{ij} &= \exp(-1 + \frac{\mu_i}{f_i} + \lambda_j) t_{ij}^{-\gamma}
\end{align*}
\]

(17)

Where \( \mu_i, \lambda_j \) are Lagrange coefficients for the constraints, which can be obtained from the iterative process depicted in Fig. 1. Substituting \( \gamma, \mu_i \) and \( \lambda_j \) into Eq. (17), the transfer demand \( h_{ij} \) can be obtained. Subsequently, the transfer demand from mode \( i \) to \( j \), \( X_{ij} \), can be obtained by

\[ X_{ij} = h_{ij} X_i \]  

(18)
**The Case Study**

The case study was conducted at the SiHui transportation terminal, a main intermodal transportation terminal in Beijing metropolitan area. The objective of the case study is to calibrate the key parameters in the entropy model by using the real-world data and apply the model to obtain the estimated transfer demands. Presently, passengers can transfer among seven transportation modes in the terminal, including suburban transit, city transit, subway, car, taxi, bicycle and walk. The total daily transfer demand in the terminal is about 300,000 passengers. The functional description of the SiHui terminal is illustrated in Fig. 2. Owing to the rapid growth in transfer demands, the SiHui terminal is undergoing critical expansion in 2010, which gives rise to the development of mathematical models to estimate transfer demands.

The case study was led by a survey to identify the critical factors to be included in the modeling approach. Equally important for the survey is to determine the relative importance of these factors, from which the weighting coefficients can be generated. The survey was conducted in June 2008 for two
weeks at the Sihui Terminal in Beijing, China. 30 undergraduate students participated in the survey. During the survey, 2109 survey questionnaires were filled by the passengers, of which 2000 were valid and used in the study to determine mainly the information shown in Table 1.

Because many factors are influential to a passenger’s mode choice behavior at transfer terminals, the main purpose of the survey was to identify the critical factors, exclude the minor factors, and determine the relative importance of these factors, i.e., the weighting coefficients. For instance, the interval travel time/walking time inside the terminal was thought to be a major factor by the researchers but found negligible from the survey and consequently eliminated from consideration in the formulation. The utility factors were narrowed down to speed, service time, fare, and level of comfort which were used to calculate the expense $t_{ij}$.

![Functional Layout of SiHui Terminal](image)

**Fig. 2.** The Functional Layout of SiHui Terminal

<table>
<thead>
<tr>
<th>Traffic mode</th>
<th>Speed (km/h)</th>
<th>Service time (min)</th>
<th>Fare ($)</th>
<th>Level of Comfort</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bicycle</td>
<td>18</td>
<td>1</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>Walk</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>City Transit</td>
<td>40</td>
<td>10</td>
<td>0.07</td>
<td>4</td>
</tr>
<tr>
<td>Suburban Transit</td>
<td>80</td>
<td>30</td>
<td>4.35</td>
<td>4</td>
</tr>
</tbody>
</table>

**Table 1. Utility Data of Each Transportation Mode**
In order to eliminate the potential negative effects caused by the different dimension and magnitude of the factors, the raw data matrix \((Y = (y_{ij}))_{n \times p}\) of Table 1 is standardized as the follows.

\[
y_{ij} = \frac{(y_{ij} - \bar{y}_j)}{s_j}
\]  

(19)

Where \(\bar{y}_j = \frac{1}{n} \sum_{i=1}^{n} y_{ij}\), \(s_j = \sqrt{\frac{\sum_{i=1}^{n} (y_{ij} - \bar{y}_j)^2}{n-1}}\), \(y_{ij}\) is the element of \(i\)th row and \(j\)th column in Table 1, \(n = 7\) is number of rows of Table 1 except “Weighting coefficient”, \(p = 4\) is number of columns of Table 1.

The travel expense \(t_{ij}\) was obtained from Eq. (19) and the results are listed in Table 2. A greater value in Table 2 implies a higher expense, with 999 being the infinite travel expense representing impossible or negligible transfers. The travel expense \((t_{ij})\) for each mode pair was then used to calibrate \(\mu_i\) and \(\lambda_j\) in the Entropy-maximizing model.

**Table 2. Travel Expense of Each Traffic Modes**

<table>
<thead>
<tr>
<th></th>
<th>Bicycle</th>
<th>Walk</th>
<th>City Transit</th>
<th>Suburban Transit</th>
<th>Car</th>
<th>Subway</th>
<th>Taxi</th>
</tr>
</thead>
<tbody>
<tr>
<td>From</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bicycle</td>
<td>999</td>
<td>0.205</td>
<td>0.119</td>
<td>999</td>
<td>999</td>
<td>0.097</td>
<td>999</td>
</tr>
<tr>
<td>Walk</td>
<td>0.146</td>
<td>999</td>
<td>0.119</td>
<td>0.202</td>
<td>0.102</td>
<td>0.097</td>
<td>0.129</td>
</tr>
<tr>
<td>City Transit</td>
<td>0.146</td>
<td>0.205</td>
<td>0.119</td>
<td>0.202</td>
<td>0.102</td>
<td>0.097</td>
<td>0.129</td>
</tr>
<tr>
<td>Suburban Transit</td>
<td>999</td>
<td>0.205</td>
<td>0.119</td>
<td>0.202</td>
<td>0.102</td>
<td>0.097</td>
<td>0.129</td>
</tr>
<tr>
<td>Car</td>
<td>999</td>
<td>0.205</td>
<td>0.119</td>
<td>0.202</td>
<td>999</td>
<td>0.097</td>
<td>0.129</td>
</tr>
<tr>
<td>Subway</td>
<td>0.146</td>
<td>0.205</td>
<td>0.119</td>
<td>0.202</td>
<td>0.102</td>
<td>999</td>
<td>0.129</td>
</tr>
<tr>
<td>Taxi</td>
<td>999</td>
<td>0.205</td>
<td>0.119</td>
<td>0.202</td>
<td>0.102</td>
<td>0.097</td>
<td>999</td>
</tr>
</tbody>
</table>

*: 999 stands for infinite travel expense.
The observed daily transfer demand was provided by the Beijing Municipal Commission of Urban Planning (Table 3). It is worth of note that the total transfer demands for the car-to-other-modes and taxi-to-other-modes are substantially low, which will have an effect on the estimation. Using the observed demand data and Eq. (3), the observed transfer probability among each mode pair was obtained and shown in Table 4. The observed gravity prior probability, \( q_{ij} \), were also calculated by the demand data and shown in Table 5.

**Table 3. The Observed Transfer Demand Distribution (Passengers/day)**

<table>
<thead>
<tr>
<th>From</th>
<th>Bicycle</th>
<th>Walk</th>
<th>City Transit</th>
<th>Suburban Transit</th>
<th>Car</th>
<th>Subway</th>
<th>Taxi</th>
<th>To</th>
<th>( x_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bicycle</td>
<td>0</td>
<td>15</td>
<td>13204</td>
<td>0</td>
<td>0</td>
<td>6700</td>
<td>0</td>
<td>19919</td>
<td></td>
</tr>
<tr>
<td>Walk</td>
<td>15</td>
<td>0</td>
<td>6238</td>
<td>30</td>
<td>70</td>
<td>3269</td>
<td>19</td>
<td>9641</td>
<td></td>
</tr>
<tr>
<td>City Transit</td>
<td>12204</td>
<td>6950</td>
<td>119584</td>
<td>7110</td>
<td>169</td>
<td>37010</td>
<td>30</td>
<td>183057</td>
<td></td>
</tr>
<tr>
<td>Suburban Transit</td>
<td>0</td>
<td>27</td>
<td>6900</td>
<td>290</td>
<td>210</td>
<td>3358</td>
<td>1930</td>
<td>12715</td>
<td></td>
</tr>
<tr>
<td>Car</td>
<td>0</td>
<td>69</td>
<td>160</td>
<td>210</td>
<td>0</td>
<td>390</td>
<td>69</td>
<td>898</td>
<td></td>
</tr>
<tr>
<td>Subway</td>
<td>6700</td>
<td>3269</td>
<td>36910</td>
<td>3360</td>
<td>390</td>
<td>0</td>
<td>20</td>
<td>50649</td>
<td></td>
</tr>
<tr>
<td>Taxi</td>
<td>0</td>
<td>19</td>
<td>26</td>
<td>1926</td>
<td>69</td>
<td>20</td>
<td>0</td>
<td>2060</td>
<td></td>
</tr>
<tr>
<td>( y_j )</td>
<td>18919</td>
<td>10349</td>
<td>183022</td>
<td>12926</td>
<td>908</td>
<td>50747</td>
<td>2068</td>
<td>( \Sigma 278939 )</td>
<td></td>
</tr>
</tbody>
</table>

**Table 4. The Observed Transfer Probability Between Each Mode**

<table>
<thead>
<tr>
<th>( h_{ij} )</th>
<th>Bicycle</th>
<th>Walk</th>
<th>City Transit</th>
<th>Suburban Transit</th>
<th>Car</th>
<th>Subway</th>
<th>Taxi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bicycle</td>
<td>0</td>
<td>0.000753</td>
<td>0.662885</td>
<td>0</td>
<td>0</td>
<td>0.336362</td>
<td>0</td>
</tr>
<tr>
<td>Walk</td>
<td>0.001556</td>
<td>0</td>
<td>0.647028</td>
<td>0.003112</td>
<td>0.007261</td>
<td>0.339073</td>
<td>0.001971</td>
</tr>
<tr>
<td>City Transit</td>
<td>0.066668</td>
<td>0.037966</td>
<td>0.653261</td>
<td>0.03884</td>
<td>0.009923</td>
<td>0.202177</td>
<td>0.000164</td>
</tr>
<tr>
<td>Suburban Transit</td>
<td>0</td>
<td>0.002123</td>
<td>0.542666</td>
<td>0.022808</td>
<td>0.016516</td>
<td>0.264098</td>
<td>0.151789</td>
</tr>
<tr>
<td>Car</td>
<td>0</td>
<td>0.076837</td>
<td>0.178174</td>
<td>0.233853</td>
<td>0</td>
<td>0.434298</td>
<td>0.076837</td>
</tr>
<tr>
<td>Subway</td>
<td>0.132283</td>
<td>0.064542</td>
<td>0.728741</td>
<td>0.066339</td>
<td>0.0077</td>
<td>0</td>
<td>0.000395</td>
</tr>
<tr>
<td>Taxi</td>
<td>0</td>
<td>0.009223</td>
<td>0.012621</td>
<td>0.934951</td>
<td>0.033495</td>
<td>0.009709</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table 5. The Observed Gravity Prior Probability**
Based on the data listed in Tables 3, 5 and Eq. (15), the value of $\gamma$ can be calculated as $\gamma = -\beta_3 = 9.3995$. $\mu_i$ and $\lambda_j$ can be obtained from the iterative process as shown in Fig. 1, with $\epsilon = 0.01$. Using Eq. (17), the forecasted distribution probability, $h_{ij}$, can be calculated and the results are shown in Table 6. Subsequently, the estimated transfer distribution can be calculated by Eq. (18) and is shown in Table 7.

Table 6. The Estimated Transfer Probability Between Each Mode

<table>
<thead>
<tr>
<th>$h_{ij}$</th>
<th>Bicycle</th>
<th>Walk</th>
<th>City Transit</th>
<th>Suburban Transit</th>
<th>Car</th>
<th>Subway</th>
<th>Taxi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bicycle</td>
<td>0</td>
<td>0.0388</td>
<td>0.7096</td>
<td>0</td>
<td>0</td>
<td>0.2511</td>
<td>0</td>
</tr>
<tr>
<td>Walk</td>
<td>0.0797</td>
<td>0</td>
<td>0.6361</td>
<td>0.0477</td>
<td>0.0034</td>
<td>0.2251</td>
<td>0.0078</td>
</tr>
<tr>
<td>City Transit</td>
<td>0.077</td>
<td>0.0336</td>
<td>0.6148</td>
<td>0.0461</td>
<td>0.0033</td>
<td>0.2176</td>
<td>0.0075</td>
</tr>
<tr>
<td>Suburban Transit</td>
<td>0</td>
<td>0.0364</td>
<td>0.6659</td>
<td>0.0499</td>
<td>0.0035</td>
<td>0.2357</td>
<td>0.0082</td>
</tr>
<tr>
<td>Transit</td>
<td>0.0986</td>
<td>0.043</td>
<td>0.7868</td>
<td>0.059</td>
<td>0.0042</td>
<td>0</td>
<td>0.0096</td>
</tr>
<tr>
<td>Car</td>
<td>0</td>
<td>0.0365</td>
<td>0.6683</td>
<td>0.0501</td>
<td>0</td>
<td>0.2365</td>
<td>0.0082</td>
</tr>
<tr>
<td>Subway</td>
<td>0</td>
<td>0.0367</td>
<td>0.6714</td>
<td>0.0503</td>
<td>0.0036</td>
<td>0.2376</td>
<td>0</td>
</tr>
<tr>
<td>Taxi</td>
<td>0</td>
<td>0</td>
<td>0.733</td>
<td>13425</td>
<td>0.0071</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 7. The Estimated Transfer Demand (passengers/day)

<table>
<thead>
<tr>
<th>From</th>
<th>Bicycle</th>
<th>Walk</th>
<th>City Transit</th>
<th>Suburban Transit</th>
<th>Car</th>
<th>Subway</th>
<th>Taxi</th>
<th>$x_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bicycle</td>
<td>0</td>
<td>733</td>
<td>13425</td>
<td>0</td>
<td>0</td>
<td>4751</td>
<td>0</td>
<td>18909</td>
</tr>
</tbody>
</table>

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The performance of the proposed model was evaluated by comparing the estimated transfer demands with the real data. It can be seen from Table 3 and Table 7 that the estimated distribution agrees well with the real data in five out of seven modes. Figs. 3-6 show the graphical demonstration of selected results. High discrepancies occurred in the car-to-other-mode and taxi-to-other-mode. The error, however, is not from the model per se, but mainly due to the error in the utility data in Table 1, which is a combined result of survey and errors coming from the calibration process of the survey data. The quality of the utility data is fundamental to the accuracy of transfer expense $t_{ij}$ (Table 2), a critical input to the entropy model. For example, the observed transfers from car and taxi to city transit are very low (160 and 26 passengers, respectively) compared to the transfers from other modes. Nonetheless, this characteristic is not reflected in the expense data in Table 2, in which there are no differences in expenses from one particular mode to the other modes. Therefore, how to enhance the quality of survey and improve the technique for calibration of the critical parameters is substantial to the accuracy of the estimation.
Fig. 3. Distribution of Transferring Passengers – From Subway to Other Modes

Fig. 4. Distribution of Transferring Passengers – From City Transit to Other Modes
Presented in this article is an entropy based modeling approach for estimation of transfer demands at an intermodal transportation terminal. The entropy model with gravity prior probability was developed and calibrated against field data. The application to a real world multimodal transfer terminal in Beijing demonstrated the effectiveness of the model, indicating the positive potential for using entropy models to estimate transfer demands at intermodal transportation terminals.
One of the challenges in estimating transfer demands is selecting a proper methodological approach that is not only theoretically sound, but also of practical values for real-life application. Gravity models, entropy models, the four-step planning process, and a network modeling approach could all possibly be applicable to the problem. However, computational efficiency makes the network modeling approach less practical, and a full planning process that takes into account the evolving regional OD and travel patterns relies greatly on demographic data that are sometimes not available.

Data for the gravity model and entropy model are relatively easier to obtain, which makes it more feasible to solve the problem. However, how to use such models will not become informative to researchers and practitioners until a real world problem is solved and documented. In this study, for example, the key factors influencing a passenger’s mode choice are narrowed down to only four parameters (speed, transfer time, cost, and level of comfort) for ease of modeling and formulation. These utility data are then used to calculate the transfer expense, which is essential to estimation of the transfer matrix.

For a better application of the entropy model, the quality of the utility data is fundamental to the final result. For example, the observed transfers from car and taxi to city transit are very low. This, however, was not reflected in the expense data and has consequently led to inaccurate result in the estimation of the car-to-other-mode and taxi-to-other mode transfers. How to enhance the quality of survey and improve the technique to calibrate the critical parameters is of primary importance to the application of entropy model. Secondly, a sensitivity analysis on the basis of extended datasets is desirable to examine how sensitive the entropy model is to the input parameters, especially, the weighting coefficients of transportation modes; thirdly, it is desirable to develop a standardized calibration process to calibrate the model according to field data.

References


