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Vehicle delay estimation at unsignalised pedestrian crosswalks with probabilistic yielding behaviour

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Unsignalised pedestrian crosswalks are commonly adopted in school and residential areas. To enhance pedestrian safety, various types of signs and crosswalk markings have been implemented, which results in motorists' probabilistic yielding behaviour and interrupted traffic flow patterns. Predicting the vehicular delay is of central importance to evaluate the level of service. However, as the interaction involves two random streams and is governed by the uncertain yielding behaviour, the analysis could be fairly challenging. In this paper, a novel method is proposed to estimate vehicular delay, which decomposes the vehicular stream into free-flow and queuing traffic. By explicitly considering the relation between the vehicular headway and the critical gap, the probability of a yielding event is derived to the expected proportion of queue formation, queue dispersion and free-flow periods. Equations of the average vehicular delay are given as a function of the vehicle volume, pedestrian volume and the yielding rate. The validation experiment using a stochastic simulation indicates that the proposed method consistently gives close estimations with absolute error less than 1 s.

Keywords: unsignalised crosswalk; yielding behaviour; vehicular delay; stochastic interrupted flow

1. Introduction

Unsignalised pedestrian crosswalks are commonly adopted in school and residential areas. Though motorists are legally required to yield to pedestrians under most circumstances for both marked and unmarked crosswalks in the USA and many European counties, the actual motorist yielding behaviour varies considerably (Fitzpatrick, Turner, and Brewer 2007) and is influenced by many factors such as pedestrian crossing treatments and roadway geometries. For example, Schroeder (2008) verified that the yielding behaviour is a multi-variable involved decision process. A yielding rate can be extracted and modelled with regression methods. Turner et al. (2006) evaluated yielding rates for different types of treatments as well as various geometric factors. In Highway Capacity Manual 2010, several field observations of yielding rates are documented regarding different crossing treatments as given in Table 1 (Transportation Research Board 2010).

The probabilistic yielding behaviour of motorists at unsignalised crosswalks results in a complicated interaction between the pedestrian and the driver. Predicting average delays to both pedestrians and drivers provides an important means for assessing the level of service (LOS) of the crossing facility. However, previous studies have focused on the pedestrian delay, while the impact of the yielding behaviour on the traffic flow was ignored. With a high yielding rate, the

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Table 1. Examples of yielding rates from field observations (Fitzpatrick, Turner, and Brewer 2007; Shurbutt, Van Houten, and Turner 2008).

Crossing treatment	Number of sites	Mean yield rate
Overhead flashing beacon (push button)	4	0.49
Overhead flashing beacon (passive activation)	3	0.67
Pedestrian crossing flags	4	0.74
High-visibility signs and markings	2	0.20
Rectangular rapid-flash beacon	17	0.81

driver may incur a serious delay given a high pedestrian volume and a queue may form or even spill over to close-spaced upstream intersections. The lack of a method to predict the vehicular delay due to the yielding behaviour is stated in the Highway Capacity Manual as a major limitation in evaluating the LOS of major street through traffic in two-way stop-controlled (TWSC) intersections (Transportation Research Board 2010). To fill this gap, this paper proposes a methodology to estimate the vehicular delay considering the probabilistic yielding behaviour for one-lane traffic.

The complexity of predicting the yielding delay is attributed from two aspects. Firstly, the yielding behaviour is uncertain, which can only be quantified by the yielding probability in a certain traffic scenario. This results in a probabilistic priority for the vehicular stream (Wei et al. 2013). If assuming the pedestrian flow holds an absolute priority, classical delay equations, for example, Akcelik's delay formula (Akcelik and Troutbeck 1991), can be applied with the estimation of the traffic capacity proposed by Griffiths (1981). However, for the probabilistic priority, an exact solution is difficult to be obtained, and conventional queuing theory may not apply. Secondly, the interaction involves two stochastic streams. If analysing the pedestrian delay, only the gap distribution of the vehicular traffic has to be considered, for which numerous models based on gap acceptance theory have been proposed (Weiss and Maradudint 1961; Cowan 1984; Guo, Dunne, and Black 2004; Hediye et al. 2014). The basic idea is to examine the gaps successively observed by pedestrians regardless their arrival headways. However, when the yielding behaviour is taken into account, arrival patterns of both pedestrian and vehicular streams have to be carefully examined.

The above two characteristics for predicting the vehicular yielding delay define a new problem, which has not been fully analysed in previous work. Some similar problems worth noticing are briefly introduced below.

The minor road traffic delay of TWSC intersections is a typical example for analysing the interaction of two traffic streams, which has been extensively studied by a number of researchers (Kyte et al. 1996; Brilon and Wu 1999; Troutbeck 1999). If regarding the major traffic stream as the server and the minor traffic as the client, the problem can be described by a single queuing system with a general or two types of service rates. However, the problem of predicting the delay due to the yielding behaviour distinguishes itself by the probabilistic priority of the two streams, in which both streams can be alternatively viewed as client or server based on the yielding decision (Boon, Van der Mei, and Winands 2011).

The limited priority merging problem is also an interesting example of the interaction of two streams (Troutbeck 1988, 1999; Bunker and Troutbeck 2003). It is considered in the limited priority that vehicles in the major stream slow down to adjust to the merging of the minor stream with a modified critical gap that can be smaller than the gap in the absolute priority merging. However, when analysing the delay occurred to the major stream, it assumes a long and consistent queue of minor-stream vehicles. The average delay of vehicles for the major stream is obtained when the merge is at the capacity.

Another entirely different methods for analysing the interaction of two streams was proposed by Helbing, Jiang, and Treiber (2005). The focus in their research was on observing and analysing the inefficient oscillation phenomenon reproduced by numerical simulations using different parameters. The solution of expected delay cannot be generally derived especially for the probabilistic priority.

This paper developed a novel method to estimate the vehicular delay with the yielding behaviour at unsignalised crosswalks. The traffic stream is decomposed into uninterrupted and queuing traffic. These two modes of traffic switch to each other when a random yielding event is triggered. The probability of this yielding event is derived by analysing three complementary yielding scenarios. The expected periods of free-flow and queuing flow can be obtained, from which the expected delay are derived as a function of the vehicular volume, pedestrian volume and yielding rate.

The rest of the paper is organised as follows. Section 2 introduces the vehicle yielding behaviour and the resulting interrupted flow patterns. Section 3 presents the modelling framework and the derivation of the model, followed by a structural analysis in Section 4. Section 5 validates the model with stochastic simulations. Section 6 concludes the paper.

2. Motorists' yielding behaviour and interrupted traffic flow pattern

In this section, headway distributions of the vehicular flow and the pedestrian flow are introduced at first, followed by discussions of the yielding behaviour and the resulting interrupted traffic flow pattern. Table 2 gives a list of variables and parameters used throughout the paper.

2.1. Vehicular and pedestrian headway distribution

Vehicular and pedestrian arrival patterns serve as the theoretical basis for analysing the interaction of two streams. In this paper, the vehicular headway is assumed to follow the shifted negative exponential distribution, while the arrival of pedestrians is a Poisson process.

The simplest headway distribution is the negative exponential distribution, which assumes that vehicles arrive at random without any dependence on the leading vehicle. But it has a serious deficiency of overestimation of small headways (Akcelic and Chung 1994; Zhang et al. 2007). Another often used model that overcomes this drawback is the shifted exponential distribution. It assumes the traffic headway cannot be smaller than a minimum threshold of t_m . Its probability density function (PDF) is as follows:

$$f(t_v) = \begin{cases} 0, & t_v < t_m, \\ \lambda_v e^{-\lambda_v(t_v - t_m)}, & t_v \geq t_m, \end{cases} \quad (1)$$

where

$$\lambda_v = \frac{q}{(1 - t_m q)} \quad (2)$$

and q is the average flow ratio; t_m is the minimum headway.

For estimating the vehicular yielding delay, the pedestrian arrival process is rather important. The most common model of pedestrians' arrival process is the Poisson model. Conceptually, applying the Poisson process does not have a significant deficiency since there is no minimum inter-arrival time or safety gap that is required for the pedestrian flow. The inter-arrival time for the Poisson process is a negative exponential distribution. It is a special case of Equation (1) where t_m equals 0. λ_p is used here to denote the arrival ratio of the pedestrian flow.

Table 2. List of variables and parameters.

<i>Related to arrival processes of pedestrians and vehicular flow</i>	
t_v	Vehicular flow headway
λ_v	Parameter of shifted exponential distribution for vehicular flow
t_m	Minimum headway of vehicular flow
q_v	Average flow rate
I	Dispersion index of the vehicle arrival process
t_p	Pedestrian headway
λ_p	Parameter of negative exponential distribution for pedestrian flow
f_v	PDF of vehicular headway
f_p	PDF of pedestrian arrival headway
<i>Related to motorists' yielding and pedestrians' crossing behaviour</i>	
δ	Safety gap (critical gap) for pedestrian crossing
M	Yielding rate
L	Probability of having waiting pedestrians who were unable to cross during previous gaps
P_y	Probability of a yielding event
P_d	Probability of a vehicle being delayed
<i>Related to vehicle delay</i>	
ρ	Lost time
$t_{qf}^{(*)}$	The length of the queue formation period. The superscript of (a) indicates the aggressive scenario, while (b) indicates the conservative scenario
$t_{qd}^{(*)}$	The length of the queue dispersion period. The superscript of (a) indicates the aggressive scenario, while (b) indicates the conservative scenario
W_q	The total delay for vehicles that arrive during queuing period
W	The total delay of vehicles in the queue
N	The number of vehicles arrive in the queue formation, queue dispersion and free-flow periods
d	Expected average vehicular delay

Note: For the negative exponential distribution, the flow rate equals to the parameter of λ_p .

2.2. Motorists' yielding behaviour

When pedestrians seek to cross the road at an unsignalised crosswalk, they observe successive gaps between the crosswalk and the coming vehicle. Only if the gap is larger than the minimum safety gap δ (the critical gap), they begin to cross. The vehicle yielding delay is triggered when the gap is smaller than the safety gap and the driver is willing to stop at the crosswalk and yield to pedestrians. As illustrated in Figure 1, pedestrian B arrived at the crosswalk when a vehicle just left the crosswalk. Since the gap between the crosswalk and the arrival of the next vehicle was larger than the critical gap, she/he was able to cross without any delay. In contrast, when pedestrian A arrived, the headway to the next vehicle was smaller than the safety gap. She/he had to wait at the crosswalk. However, in the example of Figure 1, the vehicle yielded to the pedestrian and stopped at the crosswalk. The pedestrian then began to cross the road, while the driver incurred a yielding delay.

It is assumed in this paper that all pedestrians are homogeneous, and they adopt the same constant critical gap, which is dependent on the road width, crossing speed and clearance time and can be calculated by Equations (19)–(69) in HCM 2010 (Transportation Research Board 2010). As illustrated in Figure 1, the yielding delay includes two components: the minimum delay and the lost time ρ . The minimum delay is the time the vehicle spends while waiting at the crosswalk, which equals to the crossing time. The lost time is the compensation term for the acceleration and deceleration.

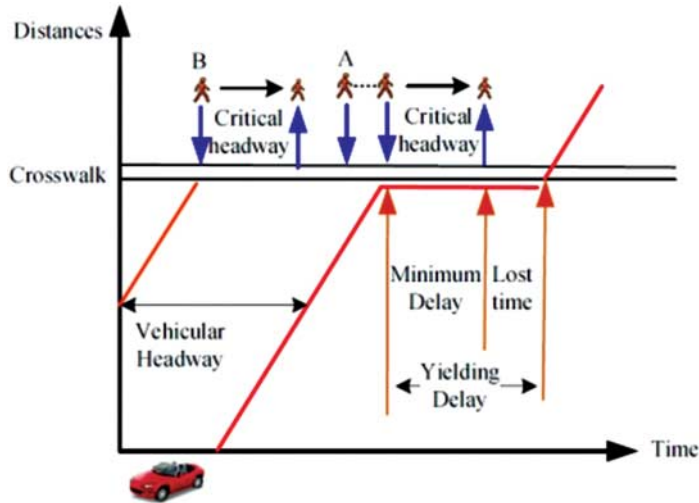


Figure 1. An example of the vehicle yielding scenario.

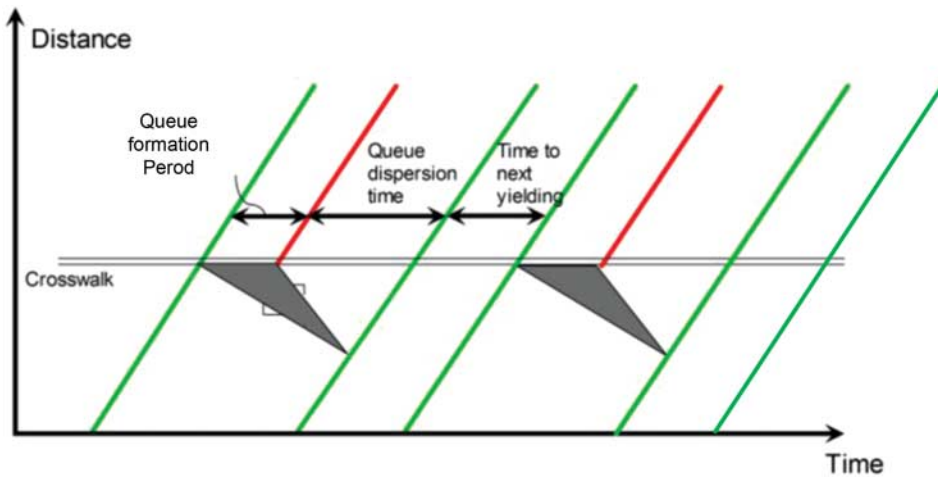


Figure 2. Illustration of traffic patterns interrupted by vehicle yielding.

2.3. The interrupted traffic flow pattern

With the yielding behaviour, the delay of the yielding vehicle may propagate to following vehicles. A queue may form at the crosswalk. This results in a stochastic interrupted flow, which randomly switches between the queuing traffic and the free flow as illustrated in Figure 2.

Two key elements have to be obtained to quantify the stochastic interrupted flow and estimate the average vehicular delay. One is the delay of the queuing vehicles during the queue formation and dispersion periods marked as the shadow area in Figure 2. Given the yielding delay of the first vehicle in the queue, the approach of the cumulative curve can be utilised to calculate the average delay for vehicles in the queue.

The other more important factor is the expected time between two random yielding events. This is essentially determined by the probability of a yielding event in the free-flow traffic, which

will be derived by explicitly considering three cases depending on whether or not the vehicular headway is larger than the critical gap and the probability of a pedestrian waiting at the crosswalk.

Two scenarios are considered for the interaction during the queue formation period. The first type of interaction is so-called ‘conservative’ scenario, where the first vehicle in the queue, which has already made a yielding decision, is willing to wait until there is no pedestrian at the crosswalk. In the other scenario (‘aggressive’ scenario), the first vehicle in the queue starts again once the first pedestrian has reached the other side of the road. Other pedestrians who arrive during the queue formation period have to wait. Compared with the ‘conservative’ case, the length of the queue formation period in the ‘aggressive’ case is obviously shorter if the pedestrian volume is relatively high. During the queue dispersion period, it is assumed that queuing vehicles would not yield to pedestrians, which is reasonable and consistent with common driving behaviour and field observations (Schroeder and Roupail 2010). Because this exhaustive clearance of the queue, the capacity of the traffic stream equals to the saturated flow rate $1/t_m$.

3. The proposed estimation method

The proposed method treats the vehicular stream as a stochastic interrupted flow. Two components including the free-flow and queuing traffic are identified by a random yielding event. By analysing three complementary cases regarding the relation between the vehicular headway and the critical gap, the probability of a yielding event is derived, which is utilised to quantify the expected proportion of free-flow and queuing traffic.

3.1. Probability of a yielding event

The probability of a yielding event is the chance that a vehicle yields to pedestrians in free-flow traffic. It determines the frequency of the transition from the free-flow traffic to the queuing traffic. The following three complementary scenarios are identified and examined depending on the arrival processes of both vehicular and pedestrian flows.

- *Case 1* In this case, the vehicular headway is larger than the critical gap. Pedestrians who fail to cross during previous gaps are able to cross the road. A yielding event may happen if a pedestrian arrives at the crosswalk while the gap before the vehicle arriving at the crosswalk is smaller than the critical gap. This case is illustrated in Figure 1.
- *Case 2* The yielding event may also happen when the vehicle’s headway is smaller than the critical headway and there are pedestrians waiting on the side of the crosswalk, who are unable to cross because previous vehicular gaps are smaller than critical headway and leading vehicles fail to yield. In this case, the following vehicle needs to make a yielding decision right after the leading vehicle passing the crosswalk. This is illustrated in Figure 3(b).
- *Case 3* The complementary yielding situation to the previous case happens when the vehicle’s headway is smaller than the critical headway and there are no pedestrians waiting on the side of the crosswalk. The driver needs to make a yielding decision only when a pedestrian arrives within its vehicular headway as shown in Figure 3(a).

Now let us derive the probability for each case. For Case 1, the probability can be easily calculated as a joint probability of having a vehicular headway larger than the critical gap and a pedestrian arriving during the gap:

$$P_1 = M \int_{\delta}^{\infty} f_v(t_v) dt_v \int_0^{\delta} f_p(t_p) dt_p, \quad (3)$$

where M is the yielding rate, f_v is the PDF of the vehicular headway, f_p is the PDF of the pedestrian headway, and δ is the critical headway.

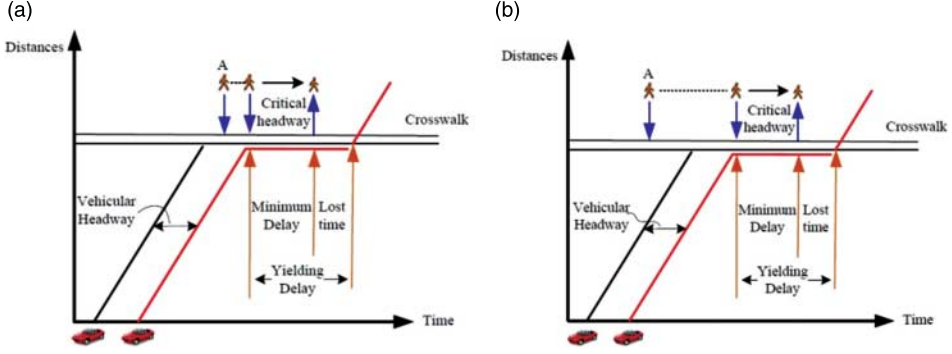


Figure 3. Illustrations of yielding delay for cases 2 and 3.

Taking the shifted exponential distribution for the vehicular headway and the Poisson arrival of pedestrian flow into Equation (3), we have

$$P_1 = M e^{-\lambda_v(\delta - t_m)} (1 - e^{-\lambda_p \delta}). \quad (4)$$

For the second case, if the vehicle's headway is smaller than the critical headway, there is a possibility that some pedestrians had already been waiting when the leading vehicle passed the crosswalk. The vehicle may need to make a yield decision once its leader passed the crosswalk. The probability of this delay will be

$$P_2 = ML \int_{t_m}^{\delta} f_v(t_v) dt_v, \quad (5)$$

where L is the probability of having pedestrians waiting from previous gaps. Considering the headway distributions, Equation (5) can further written as

$$P_2 = ML[1 - e^{-\lambda_v(\delta - t_m)}]. \quad (6)$$

An exact solution for L is attainable without considering the interaction between vehicles and the queuing effect. However, the queue formation or dispersion period is relatively longer than the consecutive headway especially with median or high traffic volume. A larger proportion of pedestrians accumulates during this period. Therefore, we provide an estimation of L as follows:

$$L = \begin{cases} 1 - e^{-\lambda_p(t_{qf}^{(a)} + t_{qd}^{(a)})}, & \text{Aggressive,} \\ 1 - e^{-\lambda_p t_{qd}^{(c)}}, & \text{Conservative.} \end{cases} \quad (7)$$

For the 'aggressive' scenario, the probability of L is the chance of having pedestrians arrive during the queue formation period $t_{qf}^{(a)}$ and the queue dispersion period $t_{qd}^{(a)}$. For the 'conservative' case, pedestrians only accumulate during the queue dispersion period $t_{qd}^{(c)}$. The calculation of $t_{qf}^{(a)}$, $t_{qd}^{(a)}$ and $t_{qd}^{(c)}$ are discussed in Section 3.2.

The last yielding case happens when the vehicular headway is smaller than the critical headway, and no pedestrian is waiting at the side of the crosswalk. In this case, the driver only needs to respond to the pedestrians who arrive within the vehicular headway. The joint probability of this

yield event is

$$P_3 = M(1 - L) \int_{t_m}^{\delta} \int_0^{t_v} f_v(t_v) f_p(t_p) dt_p dt_v, \quad (8)$$

where $\int_{t_m}^{\delta} \int_0^{t_v} f_v(t_v) f_p(t_p) dt_p dt_v$ represents the joint probability of having a vehicular headway t_v less than the critical headway δ and a pedestrian arriving during that gap. Given the headway distributions of vehicular and pedestrian flows, the probability of Equation (8) can be further calculated as

$$P_3 = M(1 - L) \left[1 - e^{-\lambda_v(\delta - t_m)} + \frac{\lambda_v}{\beta} (e^{-\beta\delta} - e^{-\beta t_m}) \right], \quad (9)$$

where $\beta = \lambda_v + \lambda_p$. Combining the above three cases, the probability of a yielding event is given as

$$P_y = M \left[1 - e^{-\beta\delta + \lambda_v t_m} + (1 - L) \frac{\lambda_v}{\beta} (e^{-\beta\delta} - e^{-\beta t_m}) \right]. \quad (10)$$

3.2. Average delay to the vehicular flow

The probability of a yielding event determines the chance of a vehicle yielding to pedestrians during the free-flow period. Once the yielding delay is triggered, it may propagate to the following vehicles and a queue may form. The delay of the queuing traffic is discussed in this section, which is used to estimate the average vehicular delay combining the probability of a yielding event.

As introduced in Section 2.2, two scenarios are identified regarding whether or not pedestrians are able to cross during the queue formation period. Let us first derive the queue formation and dispersion periods for the ‘aggressive’ scenario. In this case, the driver at the first position in the queue starts again once the pedestrian has reached the other side of the crosswalk. The length of the queue formation period is the crossing time, which equals to the critical gap δ , plus the lost time ρ as illustrated in Figure 1:

$$t_{qf}^{(a)} = \delta + \rho. \quad (11)$$

For the queue dispersion period, the expected length can be derived using the accumulative approach as shown in Figure 4 (Liu, Balke, and Lin 2008)

$$t_{qd}^{(a)} = \frac{q_v t_m}{1 - q_v t_m} t_{qf}^{(a)}, \quad (12)$$

For the ‘conservative’ case, the driver is willing to wait until there is no pedestrian waiting on the side of the crosswalk. The delay of the first vehicle in the queue is equivalent to the waiting time for a headway in pedestrian flow that is larger than the critical gap. Therefore, calculating the length of this period is quite similar to the problem of Adams’ delay (Mayne 1955). Given the Poisson arrival of pedestrian flow, $t_{qf}^{(c)}$ in the ‘conservative’ case can be determined by

$$t_{qf}^{(c)} = \rho + \delta + \frac{1}{\lambda_p} [e^{\lambda_p \delta} - (1 + \lambda_p \delta)], \quad (13)$$

And the corresponding dispersion period $t_{qd}^{(c)}$ is

$$t_{qd}^{(c)} = \frac{q_v t_m}{1 - q_v t_m} t_{qf}^{(c)}, \quad (14)$$

Now let us derive the total vehicle delay during the queue formation and dispersion periods, which includes two components. The first part is the delay incurred to the yielding vehicle at the first

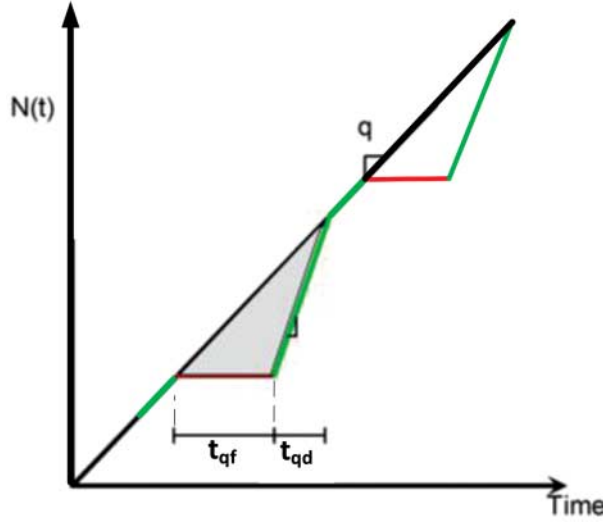


Figure 4. The cumulative curve approach for calculating the queue dispersion period.

position in the queue. This delay equals to the length of the queue formation period. The second part is the delay incurred to vehicles that arrive during the queue formation and dispersion periods. Therefore, the total delay W of the vehicles in the queue can be expressed as

$$W = W_q + t_{qf}, \quad (15)$$

where W_q denotes the total delay of vehicles that arrive during the queuing period. The derivation of W_q follows the method of McNeil (1968). The details are presented in the Appendix. The solution is expressed as

$$E(W_q) = \frac{q_v t_{qf}}{2(1 - t_m q_v)} \left[t_{qf} + t_m \left(1 + \frac{I}{1 - t_m q_v} \right) \right], \quad (16)$$

where I is the dispersion index of the arrival process and can be approximated by the following equation (Cox and Lewis 1966; Gerhardt and Nelson 2009):

$$I = \frac{q_v^2}{\lambda_v^2} = (1 - t_m q_v)^2. \quad (17)$$

Taking Equations (16) and (17) into Equation (15), the total delay can be derived as

$$E(W) = \frac{q_v t_{qf}}{2(1 - t_m q_v)} [t_{qf} + t_m (2 - t_m q_v)] + t_{qf}. \quad (18)$$

The average number of vehicles that arrive in queue formation, queue dispersion and free-flow periods is given by

$$E(N) = q_v (t_{qd} + t_{qf}) + \frac{1}{P_y}, \quad (19)$$

where $q_v (t_{qd} + t_{qf})$ is the expected number of vehicles arriving during the queue formation and dispersion periods. $1/P_y$ is the expected number of vehicles in free-flow traffic until a yielding

event occurs. Dividing $E(W)$ by the average number of vehicles $E(N)$, we finally obtain the average delay for each vehicle as

$$d = \frac{q_v t_{qf} [t_{qf} + t_m(2 - t_m q_v)] / [2(1 - t_m q_v)] + t_{qf}}{q_v(t_{qd} + t_{qf}) + 1/P_y}, \quad (20)$$

where P_y is given by Equation (10). t_{qf} and t_{qd} are given by Equations (11)–(14) for both aggressive and conservative cases.

4. Model properties

The structural properties of the model are analysed in this section to understand how the average vehicular delay changes with respect to the traffic volume, pedestrian volume and yielding rate for both ‘aggressive’ and ‘conservative’ scenarios. In this analysis, the minimum headway is set to be 2 s. The critical gap was determined according to Equations (19)–(69) in HCM 2010 (Transportation Research Board 2010), in which the lane width is set to be 12 feet, walking speed is 4 feet/s, the start time is 3 s and the loss time is set to be 5 s.

The probability of a yielding event in the three cases discussed in Section 3.1 are plotted in Figure 5 for the ‘conservative’ scenario. For vehicles with a headway larger than the critical headway, the probability of a yielding event P_1 decreases with an increase in traffic volume and a decrease in pedestrian volume, which can easily be identified from Equation (4). However, for the second case, the probability of P_2 is larger given higher vehicular and pedestrian volumes. This is actually attributed from two aspects. Firstly, Equation (6) indicates that P_2 is positively related to λ_v , the vehicular flow parameter. Secondly, the probability L of having pedestrian waiting is also larger given a higher pedestrian volume as one can identify from Equation (7). The probability P_3 in the last case is very small and decreases with an increase in pedestrian volumes since a higher pedestrian volume leads to a small probability of $(1 - L)$.

The probabilities of a yielding event for the ‘aggressive’ scenario are plotted in Figure 6. The difference compared with the ‘conservative’ scenario is due to the lower probability of L . Since

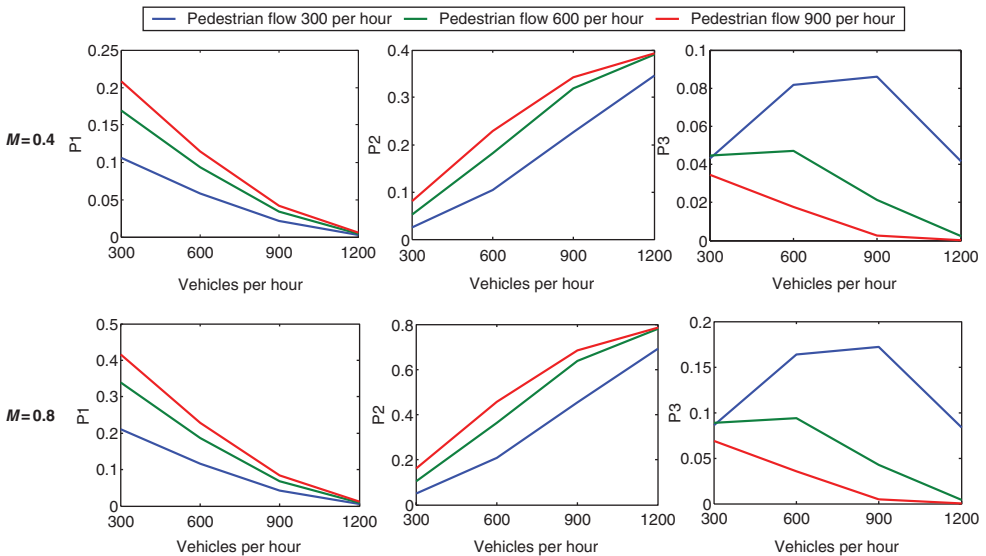


Figure 5. Probabilities of a yielding event for the ‘conservative’ case with yielding rates of 0.4 and 0.8.

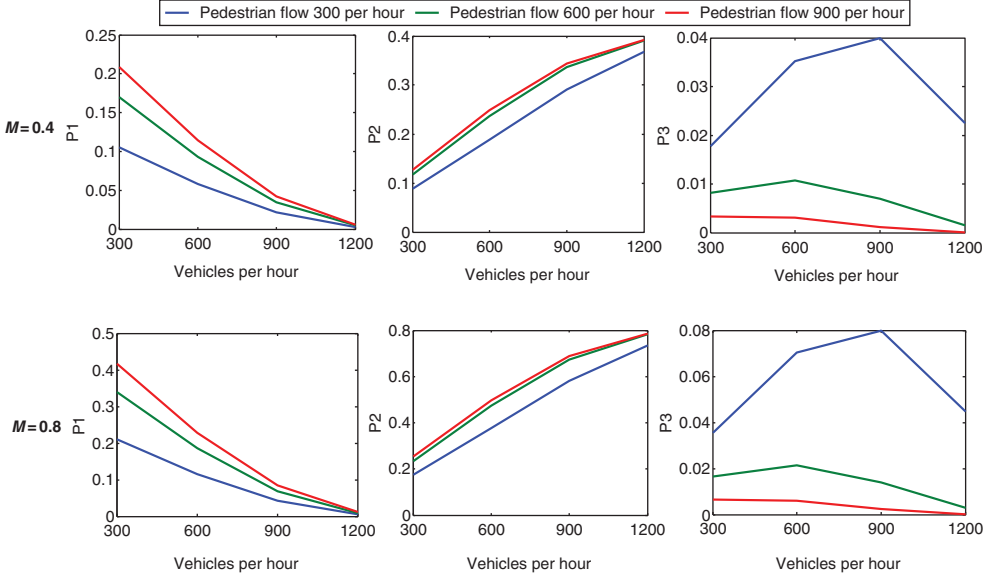


Figure 6. Probabilities of a yielding event for the ‘aggressive’ case with yielding rates of 0.4 and 0.8.

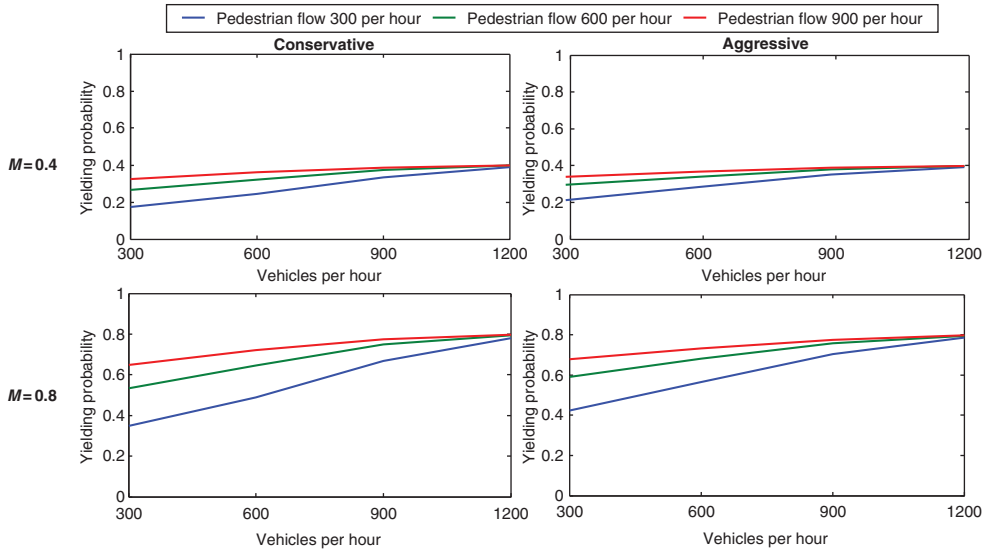


Figure 7. Yielding probabilities for ‘aggressive’ and ‘conservative’ cases.

the probability of P_1 does not depend on L , the values are exactly the same for the both scenarios. However, P_2 in the ‘aggressive’ scenario is larger compared with the ‘conservative’ case, while P_3 is smaller.

The overall yielding probabilities for different combinations of vehicular flow, pedestrian flow and the yielding rate are demonstrated in Figure 7. Several properties can be identified. Firstly, this probability increases with the yielding rate M almost linearly as one would easily identify from Equation (10). With high pedestrian and vehicular volumes, the yielding probabilities approach to the yielding rate of M . Secondly, the probability grows with an increase in vehicular or pedestrian

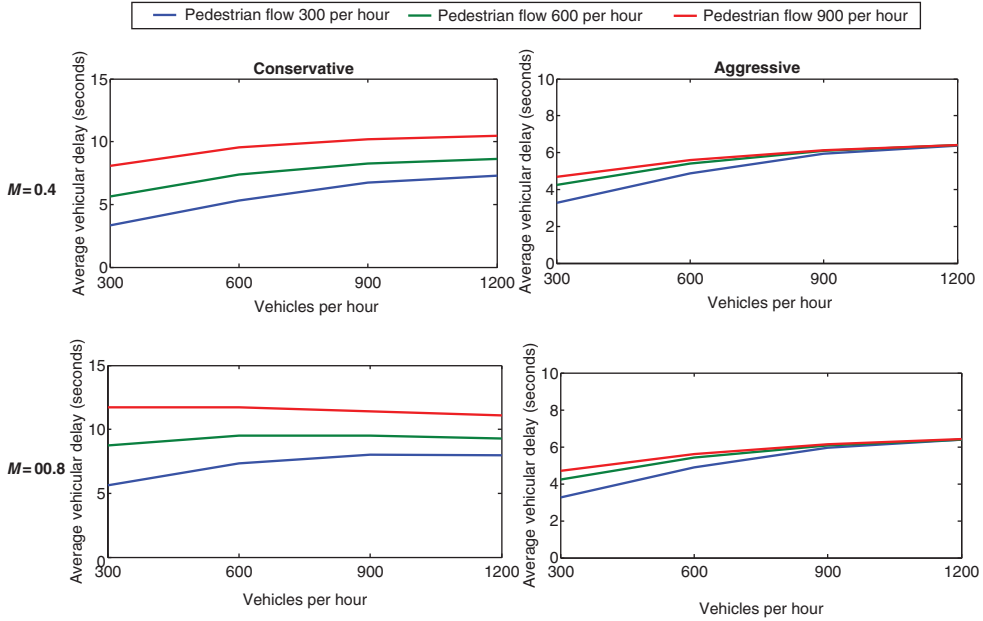


Figure 8. Average vehicular delays with different traffic and pedestrian volume for ‘aggressive’ and ‘conservative’ cases.

volume. The differences between ‘aggressive’ and ‘conservative’ are only marginal. The delay probability in the ‘aggressive’ case is slightly higher than that in the ‘conservative’ case.

The average vehicle delays are plotted in Figure 8. For the ‘aggressive’ case, the average delay converges given a high traffic volume. This is mainly because of two reasons. First, both the queue dispersion and formation periods determined by Equations (11) and (12) are independent from the pedestrian arrival rate. Secondly, the yielding probability, which determines the free-flow period, also converges as shown in Figure 7. Therefore, given high traffic volumes, the impact of the pedestrian arrival rate is almost negligible, and the vehicular delay tends to a constant. In contrast, for the ‘conservative’ scenario, a higher pedestrian volume results in a higher vehicular delay even if the traffic volume is relatively high. This is because both the queue formation and dispersion periods become longer as the pedestrian rate increases.

5. Simulation validation

Lacking a proper yielding delay collection approach, stochastic simulations that satisfy the above headway distributions are carried out to assess the validity of the analytical solution for the vehicular yielding delay.

The simulation programme utilised is a point process model, in which each vehicle is treated as a point (Grossmann 1988). This type of simulation has been used in a number of studies, particularly for validating models with stochastic arrival patterns. In our programme, vehicles arrive at the crosswalk with the successive headways satisfying the shifted exponential headway distribution. The pedestrians are generated at the side of the crosswalk according to a Poisson process. The simulation resolution is set to be 0.1 s. In every iteration, a vehicle needs to make a yielding decision according to the yielding rate if there are pedestrians waiting on the side of the crossing. A vehicle has to keep a minimum headway of 2 s in the queue dispersion period.

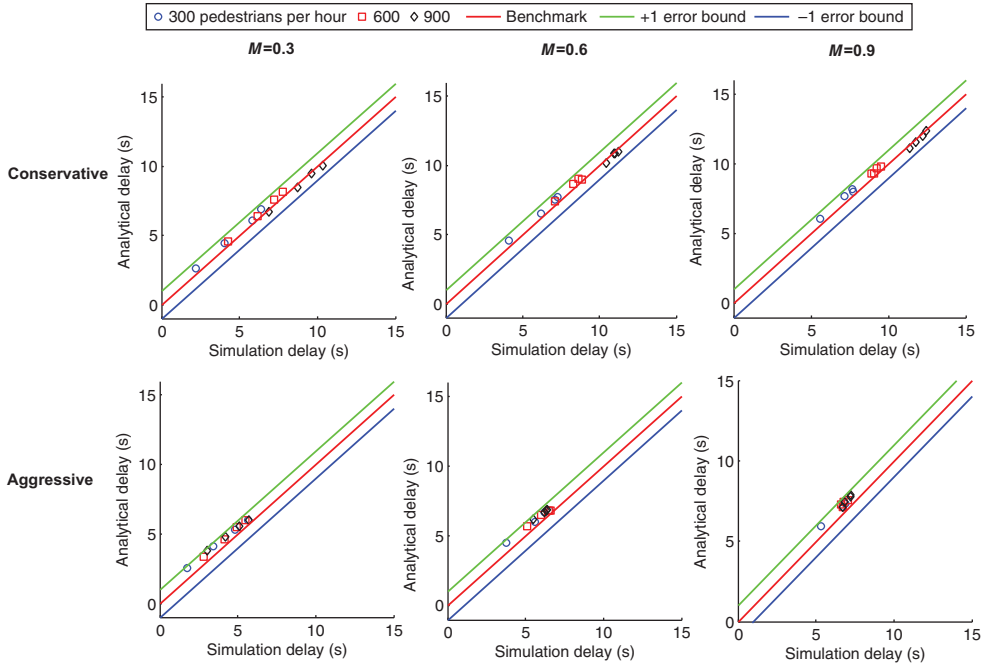


Figure 9. Average vehicular delays with different traffic and pedestrian volume for 'aggressive' and 'conservative' cases.

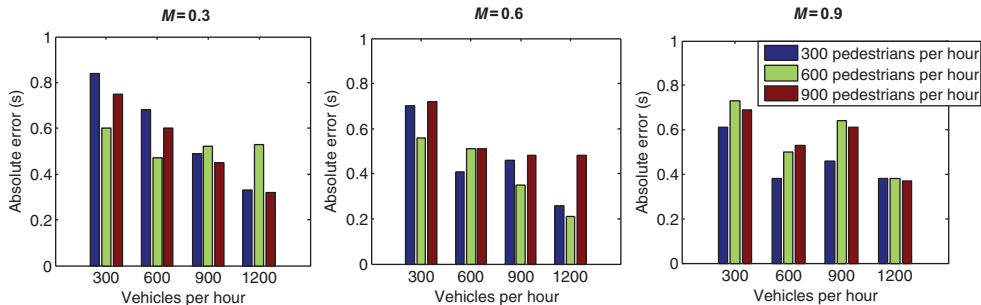


Figure 10. Absolute errors for the 'aggressive' case.

The other parameters including the start time, loss time, walking speed and the road width are the same as the settings in model property analysis.

The simulation was performed with pedestrian volumes of 300, 600 and 900 per hour and vehicular volumes of 300, 600, 900 and 1200 per hour. The yielding rates are set to be 0.3, 0.6 and 0.9. The results are averaged over 10 simulations and each simulation run for 3600 s to obtain steady states.

Results from both simulations and the proposed method are presented as diagonal plots in Figure 9. The absolute errors between the simulation outputs and the estimations are further plotted in Figures 10 and 11. Overall, in both 'aggressive' and 'conservative' scenarios, the analytical results are within 1-s deviation bounds.

For the 'aggressive' scenario, the method tends to slightly overestimate the average delay as shown in the diagonal plot. Higher yielding rates leads slightly lower errors. As the traffic volume

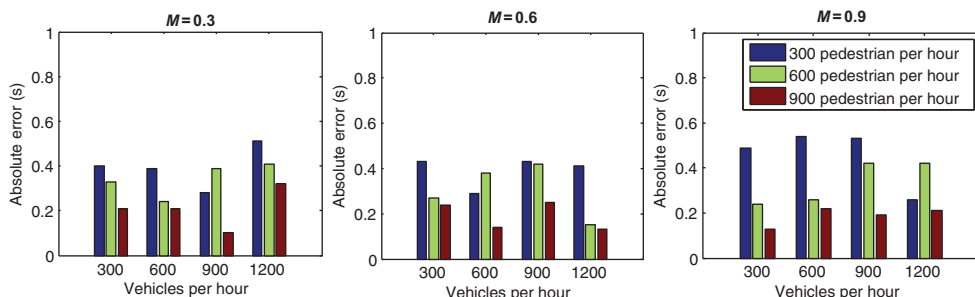


Figure 11. Absolute errors for the 'conservative' case.

increases, the absolute error decreases. The differences between the simulation and the analytical methods are mainly attributed from two aspects. Firstly, the probability of L is approximated by only taking queuing period into consideration. That may lead to slightly overestimation of the yielding probability. Secondly, the dispersion index given in Equation (17) holds only when queue period is long enough. If the traffic volume is low and the queue period is short, the variation in the traffic is amplified. However, these two approximations greatly facilitate the derivation, and only result in slightly overestimation as shown in Figures 9 and 10.

For the 'conservative' case, the absolute errors are plotted in Figure 11. Compared with the 'aggressive' case, the errors are slightly lower, most of which are below 0.6 s. This is because the queue formation period is not constant in the 'conservative' case. Using the expected value to derive the vehicle delay cannot fully capture the impact of its variation, which result in potential underestimates of the vehicle delay. But it is compensated by the overestimations due to the two approximations as explained in the 'aggressive' case. Overall, for both scenarios, the estimation model gives close predictions under different combinations of vehicular and pedestrian volumes.

6. Conclusion

The uncontrolled interactions between the pedestrian and vehicular flows at an unsignalised crosswalk leads to an interrupted vehicular flow pattern. Predicting the average vehicular delay due to the yielding behaviour is particularly important to evaluate the system performance and safety treatments.

In this paper, we proposed a method to estimate the vehicular delay, which decomposes the traffic stream into the free-flow, queue formation and queue dispersion periods. By calculating the probability of the yielding event, the distribution of the time, as well as the numbers of vehicles in each period are derived to obtain the delay probability. The average vehicular delay is then estimated based on the average delay in the queue. The stochastic simulation demonstrated that the proposed model consistently give an applicable estimation of the vehicular delay. Most absolute errors are below 1 s for the 'aggressive' case and 1.5 s for the 'conservative' case.

Extensions to multi-lane traffic can be obtained by considering a multi-lane headway distribution and cooperative yielding behaviour. In future work, a more realistic headway-dependent yielding rate and non-uniform critical gaps will be considered and integrated into the model.

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Appendix. Derivation of Equation (16)

The derivative of Equation (16) follows the method proposed by McNeil (1968). Let $N(t)$ denote the number of vehicles arriving in an interval of t . The total delay of vehicles during the queue formation period is given by:

$$W_{qf} = \int_0^{t_{qf}} N(t) dt \quad (A1)$$

and its expectation is

$$E(W_{qf}) = \frac{1}{2} q_v t_{qf}^2, \quad (A2)$$

Let T denote the time when the queue has cleared and $Q(t)$ denote the number of vehicles in the queue at time t and. $E[Q(t_{qf})] = q_v t_{qf}$. The total delay of vehicles during the queue dispersion period is

$$W_{qd} = \int_{t_{qf}}^T Q(t) dt. \quad (A3)$$

McNeil (1968) has shown that

$$\begin{aligned} E(W_{qd}) = & \frac{t_m}{2(1 - t_m q_v)^2} \{ (1 + q_v t_m I - q_v t_m) (E[Q(t_{qf})] - E[Q(T)]) \\ & + (1 - q_v t_m) (E[Q^2(t_{qf})] - E[Q^2(T)]) \}, \end{aligned} \quad (A4)$$

where I is the dispersion index given by $I(t) = \text{var}(N(t))/E(N(t))$. At the end of the queue formation period, the first vehicle would start again and leave the queue. Therefore, we have $E[Q(t_{qf})] = E[N(t_{qf})] = q_v t_{qf}$. Since the queue will always be cleared before the next yielding event, $Q(T)$ equals to zero. Hence, Equation (A4) can be simplified as

$$E(W_{qd}) = \frac{t_m}{2(1 - t_m q_v)^2} \{ (1 + q_v t_m I - q_v t_m) q_v t_{qf} + (1 - q_v t_m) (q_v^2 t_{qf}^2 + I q_v t_{qf}) \}. \quad (A5)$$

Together with Equation (A2), the total delay of vehicles arriving during the queue formation and dispersion period:

$$\begin{aligned} E(W_q) = & \frac{t_m q_v \{ (1 + q_v t_m I - q_v t_m) t_{qf} + (1 - q_v t_m) (q_v t_{qf}^2 + I t_{qf}) \}}{2(1 - t_m q_v)^2} + \frac{1}{2} q t_{qf}^2 \\ = & \frac{t_{qf} t_m q_v + t_{qf}^2 q_v^2 t_m}{2(1 - t_m q_v)} + \frac{t_{qf} t_m q_v I}{2(1 - t_m q_v)^2} + \frac{q_v t_{qf}^2 - t_{qf}^2 q_v^2 t_m}{2(1 - t_m q_v)} \\ = & \frac{t_{qf} t_m q_v}{2(1 - t_m q_v)} + \frac{t_{qf} t_m q_v I}{2(1 - t_m q_v)^2} + \frac{q_v t_{qf}^2}{2(1 - t_m q_v)} \\ = & \frac{q_v t_{qf}}{2(1 - t_m q_v)} \left[t_{qf} + t_m \left(1 + \frac{I}{1 - t_m q_v} \right) \right]. \end{aligned} \quad (A6)$$